

Topology and Three-Dimensional Electromagnetics

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Abstract

By considering problems which arise in both computational electromagnetics and the topology of three-dimensional manifolds, we will develop a distinction between the “intuitive but uncomputable” and the “unintuitive but computable.”

We first consider issues of decidability and algorithmic complexity in low dimensional topology and then turn to the modelling of electromagnetic systems by simple electrical circuits. That is, the extraction of “topological degrees of freedom” from models based on Maxwells equations. This amounts to the computation of the period matrix associated with harmonic representatives of relative cohomology classes. This in turn, is “merely commutative algebra”. However additional algorithmic, mechanical, and user-interface constraints quickly pull us into nonabelian algebraic structures (homotopy). This forces us to develop computational and intuitive understandings of the Hurewicz map (from homotopy to homology) and, in particular, of the commutator subgroup of the fundamental group. In this way, constraints arising in the computer aided design of electromagnetic devices, expose the distinction and interplay between the intuitive and computable aspects of three-dimensional topology.