The chiral de Rham complex, orbifolds, elliptic genera, and automorphic forms

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Abstract

Mathematicians have devoted much effort towards rigorously describing various aspects of quantum field theory (QFT). One mathematical object that captures the algebraic content of (some) QFT's is a vertex algebra. Given a complex manifold X, one can attach to it a QFT known as the "supersymmetric sigma-model with target X", denoted $\Sigma(X)$. One would of course like to have a geometric construction of the vertex algebra underlying $\Sigma(X)$. A candidate for such a construction is the chiral de Rham complex introduced by Malikov, Schechtman, and Vaintrob in 1998. It is a sheaf of vertex algebras on X, denoted Ω^{ch}_X, whose cohomology captures interesting "stringy" invariants related to $\Sigma(X)$.

This talk will introduce vertex algebras, the sheaf Ω^{ch}_X, and its relation with algebraic loop-spaces. I will discuss joint work with Edward Frenkel extending the construction of Ω^{ch}_X to orbifolds. I will describe how it captures stringy geometric invariants of orbifolds such as Chen-Ruan orbifold cohomology and orbifold elliptic genera. Finally, I will discuss joint work with Anatoly Libgober on discrete torsion, and how beautiful generating functions that arise in this context seem to have a deep relation with the theory of automorphic lifts.