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Title: Geometric origin of group invariant soliton equations and integrable curve flows

Abstract: The theory of integrable soliton equations displays many deep links to differential geometry, particularly as found in the study of geometric curve flows. In this talk, I will show how group-invariant multicomponent soliton equations of mKdV and sine-Gordon type along with their bi-Hamiltonian integrability structure arise in a geometrical fashion from studying non-stretching flows of curves in Riemannian symmetric spaces G/H. A key idea is the natural construction of a parallel moving frame formulation for non-stretching curves, using the Klein geometry attached to the Lie algebra structure of symmetric spaces.

The results give a broad generalization of the well known mKdV/sine-Gordon hierarchy of scalar soliton equations, whose geometric origin is connected with flows of space curves on the 2-sphere as found in the work of Goldstein& Petrich (Phys. Rev. Lett. 67 (1991), Doliwa& Santini (Phys. Lett. A 185 (1994), and Mari Befa, Sanders, Wang (J. Nonlin. Sci. 12 (2002)). In particular, the two known vector generalizations of the mKdV and sine-Gordon equations (and their bi-Hamiltonian integrability structure) will be shown to originate from the geometry of non-stretching curve flows in $SO(N+1)/SO(N) \simeq S^N$ and $SU(N)/SO(N)$ which exhaust all examples of Riemannian symmetric spaces $G/SO(N)$ for compact simple Lie groups G. Moreover the curve flows themselves will be seen to correspond to analogs of non-stretching Schrodinger maps on $G/SO(N)$ in the mKdV case and non-stretching wave maps on $G/SO(N)$ in the sine-Gordon case.