Prym varieties and the Schottky problem for cubic threefolds

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Abstract
A theorem of Mumford’s states that for a smooth cubic threefold $X$, the intermediate Jacobian $J_X$ is a principally polarized abelian variety of dimension $5$ whose theta divisor has a unique singular point, which has multiplicity three. This talk describes joint work with R. Friedman, in which we prove a converse: if $A$ is a principally polarized abelian variety of dimension $5$ whose theta divisor has a unique singular point, which has multiplicity three, then $A$ is the intermediate Jacobian of a smooth cubic threefold. The method of proof is to view $A$ as a generalized Prym variety and to use this description to analyze the singular points of the theta divisor. Along these lines, I will also discuss recent work which gives a sharp upper bound on the multiplicity of a point on the theta divisor of an irreducible principally polarized abelian variety of dimension at most five.