Langlands transform and Painlevé equations

Let X be a smooth compact algebraic curve over complex numbers (a.k.a. a Riemann surface) and G be a reductive group (for example $SL(n)$). The (mostly conjectural) Langlands transform is an equivalence between some categories associated to the moduli space of principal G-bundles on X and the moduli space of $G^\vee$-bundles with connections. Here $G^\vee$ is the so-called *Langlands dual group of* G.

I shall explain in details what the above equivalence means and (maybe) discuss the relation with the classical Langlands correspondence. Then I will talk about the "Painlevé–VI case" proved by D. Arinkin, and about other cases of Langlands transform proved recently by Arinkin and myself.