We consider an extension of the non-compact Calabi-Yau theorem of S. Bando, R. Kobyashi, G. Tian, and S. T. Yau. A similar construction of non-compact complete Ricci-flat metrics on quasi-projective varieties $Y = X \setminus D$ with $\alpha[D] = c_1(X), \alpha > 1$ is given, where $D$ is a smooth divisor which in this case possibly does not admit a Kähler-Einstein metric. This construction produces examples of Ricci-flat Kähler metrics on manifolds $Y = X \setminus D$ where $D$ is toric. The proof makes use of a result of A. Futaki, H. Ono, and G. Wang that the link $S$ in the normal bundle of $D$, this case a toric Sasaki manifold, admits a Sasaki-Einstein metric.