Abstract: I will first review two known constructions involving framed links in 3-manifolds $M$.

1) If $M = \mathbb{R}^3$, one can consider the Jones polynomial of the link: this is an element of $\mathbb{Z}[q, 1/q]$.

2) If $M = C \times \mathbb{R}$ for a surface $C$ equipped with an ideal triangulation, one can consider the ”quantum trace” associated to the link, introduced by Bonahon-Wong: this is an element of a certain noncommutative algebra associated to an ideal triangulation of $C$.

I will explain a geometric perspective on these constructions in the language of spectral networks. This perspective leads to a further generalization (work in progress with Fei Yan), which associates an invariant to a framed link in a more general 3-manifold $M$. The invariant is valued in a certain abelian group, the ”$GL(1)$ skein module” of a double cover of $M$. The construction can be viewed as a q-deformation of the abelianization of flat $GL(2)$-connections over $M$, and has relations to Gromov-Witten theory and to quantum field theory; I will describe these points as time permits.

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