Intersection Cohomology $\mathcal{D}$-Module and Algebraic Residues

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Abstract

Suppose X is a variety over a field k of characteristic 0, embedded in a smooth variety Y. According to Brylinski-Kashiwara there is a unique simple $\mathcal{D}_Y$-module $\mathcal{L}(X,Y) \subset \mathcal{H}^d_X \mathcal{O}_Y$, where $d = \operatorname{codim}(X,Y)$. If $k = \mathbf{C}$ then the De Rham complex $\mathrm{DR}\mathcal{L}(X,Y)$ is the intersection cohomology sheaf of X. Vilonen found a complex-analytic description of $\mathcal{L}(X,Y)$ when X is a locally complete intersection with isolated singularities. The explicit construction of Grothendieck's residue complex allows a description of $\mathcal{L}(X,Y)$ in terms of algebraic residues, when X is a curve. An outline of the proof of this result is given. For the case $\dim X > 1$ we have a conjectured formula.