Folding and coloring problems in mathematics and physics

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Abstract

I formulate various (planar) lattice folding problems arising in the context of statistical physics of surfaces. A folding is a map from the lattice to the d-dimensional space preserving its faces. I'll show how one can define thermodynamical quantities such as the folding entropy, as suitable limits involving the number of distinct folded configurations of the surface, and present the case of the two-dimensional folding of the triangular lattice, and many more. Most of these problems may be mapped onto an equivalent vertex/face or edge coloring problem for the lattice. In a second step, I address the folding problem of random surfaces, in the form of graphs with arbitrary connectivity. In particular, I'll show how the vertex tricoloring problem of random triangulations is solved by a simple matrix integral. Finally, I'll address the folding of one-dimensional (polymer) chains. I'll describe the underlying algebraic structure of the problem, and suggest a general scheme to investigate the case of arbitrary dimensional objects.