Pseudoholomorphic curves in symplectisations and some global problems in contact geometry

Casim Abbas
Department of Mathematics
Courant Institute for the Mathematical Sciences

Abstract

In the few last years significant progress has been made in the study of the Reeb-dynamics of a three dimensional contact manifold and its implications on the topology of the underlying three-manifold. While the focus of past research was on periodic orbits of the Reeb vectorfield, this seminar talk is dealing with a different class of trajectories: The so-called characteristic chords. These are orbits of the Reeb vectorfield that hit a given Legendrian knot at two different times. In the case of periodic orbits the main tools were pseudoholomorphic curves in the symplectisation of the contact manifold with domain being a punctured Riemann surface without boundary, while in the case of characteristic chords the domain will have boundary and also punctures on the boundary. A free boundary condition will also be imposed. I will discuss existence results for characteristic chords, which is related to a conjecture of V.I. Arnold in contact geometry. A new method developed here is a version of a classical result by E. Bishop about local fillings of a real surface in $\mathbf{C}^2$ near an isolated elliptic complex tangency. The existence question for characteristic chords is significant for a new invariant of contact manifolds, called "Contact Homology".