The Decomposition Theorem: Hilbert schemes and Heisenberg algebras

Mark de Cataldo
Department of Mathematics
Harvard University

Abstract

Given a complex algebraic surface X, the associated Hilbert scheme is a collection of projective varieties which parameterizes the subvarieties of X. The rational cohomology groups of the Hilbert scheme are naturally the standard representation of a certain infinite dimensional graded Lie algebra, called the Heisenberg/Clifford algebra. In the hyperkähler case the character formulas of these representations are modular forms.

The cohomology groups mentioned above, though transcendental in nature have been computed using the purely algebraic method of reduction to finite fields (Weil Conjectures).

In joint work with L. Migliorini we give a new, simpler and geometric method, suggested by the presence in the picture of the Heisenberg algebra, to calculate these cohomology groups. In particular, the method works for not necessarily algebraic surfaces.