Mathematical Physics seminar at BU

Potts Model, O(n) non-linear Sigma-models and Spanning Forests

Andrea Sportiello
(Universita' degli Studi di Milano)

In modern language, Kirchhoff Matrix-Tree Theorem (of 1847) puts in relation the (multivariate) generating function for spanning trees on a graph to the partition function of the scalar fermionic free field (a.k.a. 'ghost theory'). A trivial corollary concerns rooted spanning forests and the massive perturbation of the free field.

We generalize these facts in many respects. In particular, we show that a fermionic theory with a 4-fermion interaction gives the generating function for unrooted spanning forests. Remarkably, this theory coincides with the perturbative theory originated from a non-linear Sigma-model with OSP(1—2) symmetry, which, in Parisi-Sourlas correspondence, is expected to coincide with the analytic continuation of O(n) model to n –¿ -1.

In two dimensions, and on a regular lattice, this theory has a RG flow from the fixed point of spanning trees (a logarithmic CFT with c=-2) towards the high-T atomic-forest limit. The c=-2 CFT is marginally unstable, so the model presents asymptotic freedom.

The relation between spanning forests and the fermionic theory can be proven directly. However, the underlying OSP(1—2) symmetry leads to the definition of a subalgebra of Grassmann Algebra (the scalars w.r.t. global rotations), with a set of surprising properties that quite simplify all the proofs.

With some effort we can also generalize the whole derivation to a family of theories with OSP(1—2n) symmetry, with n a positive integer.