Graduate Exams in Mathematics and Statistics - 2009–2010

This document explains the system of exams graduate students must pass to obtain an MA or PhD degree from our department. This system has different requirements for students in Probability and Statistics and for students in Pure and Applied Mathematics.

Please inform the Director of Graduate Studies (DGS), Professor Paul Blanchard, as soon as possible of your chosen track (Probability and Statistics or Pure and Applied Mathematics), of your intended exam schedule and your intended graduation date. Also, please bring to the DGS any questions you may have about the exam system or any other aspects of graduate studies.

I. Graduate Exams in Statistics

This is an introduction to the exam system for graduate students in statistics. The exams are different for Masters and for PhD students. Please read carefully.

Statistics exams for MA students. All MA students must take the following two exams: the MA Theory exam and the MA Applied exam. These are offered every April.

The MA Theory exam covers the material from MA581 Probability, MA582 Mathematical Statistics, and MA583 Introduction to Stochastic Processes. There are two questions from each course. Students are required to answer four questions, including at least one question from each area.

The MA Applied exam covers the material from MA575 and MA576, Applied Regression and Analysis of Variance I and II. There are three questions from each course. Students are required to answer four questions, two from each course.

Statistics exams for PhD students. First, all students in the PhD program must take the following two-semester sequences: MA779 and MA780, MA781 and MA782, and MA750 and MA751.

Then, to qualify a student to begin work on a PhD dissertation, she/he must pass two of the following three exams at the PhD level: probability, mathematical statistics, and applied statistics. The probability and mathematical statistics exams are offered every October and the applied statistics exam is offered every April.

(1) *PhD exam in Probability:*

This exam covers the material covered in: MA779 and MA780, Probability Theory I and II.

(2) *PhD exam in Mathematical Statistics:*

This exam covers the material covered in: MA781 Estimation Theory and MA782 Hypothesis Testing.

(3) *PhD exam in Applied Statistics:*

This exam covers the same material as the MA Applied exam and is offered at the same time, except that in order to pass it at the PhD level a student must correctly solve five problems, with at least two from each category.

A student who does not pass the graduate exam the first time will have a second chance. All questions about graduate exams in statistics should be directed to Professor Eric Kolaczyk, Director of the Program in Statistics.

II. Graduate Exams in Pure and Applied Math

For graduate students in Pure and Applied Mathematics, there are two exams: the preliminary exam (known to the Registrar's Office as "Comprehensive Exam"), which a student must pass to obtain an MA; and the qualifying exam, which a student takes roughly when she/he is ready to begin work on a dissertation topic. These exams, along with the relevant deadlines, are explained in more detail below.

The Preliminary Exam

Every MA student must pass this exam at the Master's level within two years of entrance into the program. Every PhD student admitted to the post-bachelor's PhD program must pass this exam at the PhD level within two years of entrance into the program. The exam is given on the first Saturday of April of each academic year, and all students must take the exam each April until they have passed. A student who does not pass the preliminary exam at the desired level will have another chance in the following year.

There are two levels on the exam: (i) a Master's pass indicates that the student has demonstrated a firm understanding of the material. Achieving a Master's pass and fulfilling the 8-course requirement are the requirements for an MA. (ii) A PhD pass indicates that the student has a command of the material necessary to begin studying the more difficult topics on the qualifying exams, and it is a prerequisite to being admitted to candidacy status in the PhD program.

The exam covers the contents of three semesters of calculus (through vector calculus), elementary differential equations, analysis on the real line, linear algebra, and elementary abstract algebra. At BU, these topics are covered in MA123–124–225, MA226, MA242, MA511–512 and MA541–542. Please find a detailed syllabus below. Copies of old preliminary exams are available both in the form of PDF files linked from the graduate program webpage and in the form of paper copies in a file folder in the main office. Feel free to either download the PDF files or to make copies of these paper copies.

Syllabus for Preliminary Exam

Morning Exam (3 hours)

I. Calculus. Basic definitions and theorems about continuity, differentiability, and integration, techniques of integration and differentiation, trapezoidal rule, Taylor approximations, infinite series, Taylor series, polar coordinates, vector geometry, vector calculus through Green's, Stokes', and Divergence Theorems in $\mathbf{R}^3$.
Suggested Reference: Stewart, *Calculus: Concepts and Context*, especially Chapter 8 (infinite series and Taylor series) and Chapters 9–13 (vector geometry and vector calculus).

II. Differential Equations. (a) First order differential equations: analytical methods (separable equations, linear equations), qualitative ideas (equilibria, slope fields, Euler's method). (b) Second order equations and first order systems: first order autonomous systems (vector fields, phase planes interpretation of graphs of solutions); linear constant coefficient second order equations, harmonic oscillator (mass spring), relationship to first order systems. (c) Analytical methods for linear first order systems and constant coefficient linear second order equations: eigenvalues, eigenvectors, relationship between characteristic polynomial for second order equation and linear system, classification of linear systems by eigenvalues. (d) Forced mass-spring system, resonance.
Suggested Reference: Blanchard, Devaney, Hall, *Differential Equations*, Chapters 1, 2, 3, 4.1–4.4, 5.1–5.2.

III. Analysis on $\mathbf{R}$ and $\mathbf{R}^n$. Topology of metric spaces, convergence of sequences and series, continuity of functions, continuous functions on compact and on connected sets, properties of derivatives, the Riemann integral, sequences and series of functions (uniform convergence), special functions (exponential, logarithmic, trigonometric) and their power series, functions of several variables (inverse and implicit function theorems, Jacobians and determinants, chain rule).
Suggested Reference: Rudin, *Principles of Mathematical Analysis,* 3^{rd} edition, Chapters 1–9.

Afternoon Exam (3 hours)

I. **Linear Algebra.** Solving systems of linear equations. Vector spaces (bases, linear independence, rank of a matrix). Inner product spaces (Gram-Schmidt, Cauchy-Schwarz). Linear transformations (kernel, range, associated matrices). Determinants. Eigenvalues, eigenvectors. Diagonalizable matrices, e.g., real symmetric matrices.
Suggested Reference: Lay, *Linear Algebra and Its Applications*, 3^{rd} edition, Chapters 1, 2.1–2.3, 3, 4.1–4.7, 5.1–5.5, 6.1–6.4 and 6.7, 7.1.

II. Abstract Algebra. (a) Basic properties of groups (excluding Sylow Theorems), homomorphisms and the First Isomorphism Theorem, normal subgroups and quotient groups. (b) Ring theory: ideals and quotient rings, polynomial rings, factorization of integers and polynomials, structure of abelian groups. (c) Vector spaces over a field, bases and linear independence, dual spaces. (d) Linear transformations: expressions in terms of a bases, characteristic polynomial, canonical forms (up to understanding of Jordan canonical form), trace and determinants. (e) Fields: field extensions, finite fields, fundamental theorem of algebra, Galois groups (definition and basic examples only, e.g. Galois groups of quadratic, cubic polynomials, roots of unity)
Suggested Reference: Artin, *Algebra*, Chapters 1–4; 10; 11.1; 12.1–7; 13.1–3,5,6,9.

The Qualifying Exam

The qualifying exam tests the students' knowledge of two basic fields of mathematics at the graduate level as well as his/her knowledge in one more specialized topic. Passing the qualifying exam indicates that the student is ready to find a dissertation topic (and a dissertation advisor).

The basic fields the student may choose from include, but are not limited to, real and complex analysis, algebra, topology, differential geometry, dynamical systems, applied mathematics, numerical analysis, operations research, probability, and logic. The student should first decide on two fields of concentration, and then ask three faculty members with expertise in the fields to be the examination committee. The student and the faculty members will then decide upon a third specialty topic. For example, a student could take a qualifying exam in dynamical systems and differential geometry, with a specialty in Hodge theory. All PhD students must choose their committee during the first semester after the completion of their preliminary exam. It is expected that one of the committee members will become the student's dissertation advisor.

The committee and the student should formulate a written syllabus for each area of the qualifying exam. The syllabus for each of the basic areas should cover one year of course work at the 700 level, although the student and the committee may adjust the syllabus. The content of the specialty topic is decided upon by the student and the committee.

The qualifying exam is an oral exam, typically two to three hours in length. It must be taken within two years of the beginning of the student's first 700 level course, and a student who does not pass the exam may retake it one more time. In any case, the exam must be passed by the end of the student's third year of graduate study (including any 500 level work). We emphasize that the qualifying exam should be taken by the beginning of the third year at the latest, in order to leave enough time for a second attempt, if necessary.

Please contact the Director of Graduate Studies, if you have any questions or concerns about the exam system in pure and applied mathematics.

III. Language Exam

- Every student in the PhD program must take a written language exam in one of the accepted languages in mathematics, French, German or Russian. The student will select a faculty examiner who will determine the content and format of the exam (typically a take-home translation).
- This exam may be taken at any time.
- Foreign students whose first language (through high school) was different from English may be exempted from the language requirement.