1) (15 points)
   a) Find \( \gcd(4237, 4814) \).
   b) Find all solutions to
       \[ 4237x + 4814y = \gcd(4237, 4814) \].

2) (10 points) Suppose \( \gcd(a, b) = 1 \) and \( a \mid bc \).
   a) Show \( a \mid c \).
   b) Give an example to show that if \( \gcd(a, b) \neq 1 \) then \( a \mid bc \) does not imply that \( a \mid c \).

3) (15 points) \( 2^{2820} \equiv 1 \pmod{2821} \).
   a) Compute \( 3^{2820} \pmod{2821} \) using the method of successive squaring. You should double-check your answer.
   b) Can you conclude from the above calculations that 2821 is composite? Why or why not?
   c) Can you conclude from the above calculations that 2821 is prime? Why or why not?

4) (15 points)
   a) How many primes \( p \) satisfy \( p \equiv 497 \pmod{502} \)? Explain your answer.
   b) If you think your answer to a) is not zero, find one and explain why it is prime.

5) (15 points) A group of 15 bandits steal a pile of gold coins. When they try to divide the coins into equal piles there are 9 left over. They fight over the extra coins and one bandit is killed. They divide the coins equally again and there are now 5 left over. They fight again and another bandit is killed. When they divide the coins equally again there are none left over.
   a) Find the smallest number of coins there could have been.
   b) Explain how you know your answer in a) is the smallest possible number.
   c) What is the next largest number of coins there could have been? Explain your answer.

6) (15 points) Show that if \( n \) is a positive integer having \( k \) distinct odd prime divisors, then \( \phi(n) \) is divisible by \( 2^k \).

7) (15 points) Explain why for any positive integer \( n \) it is always possible to find \( n \) consecutive integers, none of which is prime.

**EXTRA CREDIT:** Work on this only after you have completely finished the above. Extra credit will be given only for a correct answer that is fully and carefully explained.

Is there a positive integer \( n \) such that \( \phi(n) = 14 \)? Explain why or why not.