\> \textbf{read `/home/tkohl/maplestuff/secant`; }
\>
\>$f := x \rightarrow x^2$
\>$d := h \rightarrow \frac{f(2 + h) - f(2)}{h}$

\> # Here we’re examining $f(x) = x^2$ at the point $(2, 4)$ to demonstrate why the slope at the point $(2, 4)$ is 4,
\>
\> # or, more to the point, how the slope of the secant line approaches that of the tangent line as $h \rightarrow 0$.

\> # As I claimed already, the slope of the tangent line to the graph of $y = x^2$ at the point $(2, 4)$ is 4,
\>
\> # since the derivative is $2x$, letting $x = 2$ gives the value of 4 for the slope.

\> # Let’s look at the secant line vs. tangent line picture when $h = 2$
\> (i.e. $Q = (4, 16)$ )
\> \textbf{drawit(2);}
What is the difference quotient here? (i.e. the slope of the secant line $PQ$)

This is not surprising since the secant line and tangent line have clearly different slopes just from looking at the picture.

Let's look at the secant line vs. tangent line picture when $h=1.5$ (i.e. $Q=(3.5,12.25)$)

drawit(1.5);
> # What’s the difference quotient now?
> d(1.5);
>
> 5.500000000
>
> # OK, a little closer to 4 than before but still not quite there yet.
>
> # Let’s look at the secant line vs. tangent line picture when h=1 (Q=(3,9))
> drawit(1);
> # What’s the difference quotient now?
> d(1);

> # Let’s keep going...
> # Let’s look at the secant line vs. tangent line picture when h=0.5
> ( Q=(2.5,6.25) )
> drawit(0.5);
What’s the difference quotient now?

4.50000000

Notice how the secant line compares with the tangent line, there slopes are very close.

Let’s look at the secant line vs. tangent line picture when h=0.1

Q=(2.1, 4.41)
> # What's the difference quotient now?
> d(0.1);

> 4.100000000

> # Again, this definitely going somewhere, the smaller we make h, (i.e. the closer Q is to P) the more the secant line
> # comes into position with the tangent line.
>
> # Let's look at the secant line vs. tangent line picture when
> h=0.01 (Q = (2.01,4.0401);
> drawit(0.01);
Let's look at the secant line vs. tangent line picture when $h=0.001$

drawit(0.001);
> #Now let’s put this together and see the action as h goes from 2 to 0!
> anim(2);
> QP = (2, 4)
The connection is clear, as \( h \to 0 \), \( Q \to P \) and the secant line becomes the tangent line.

Here, \( d(h) \), the difference quotient is

\[
\frac{(2 + h)^2 - 4}{h}
\]

Let’s simplify this...

\[
4 + h
\]

Ah! so it’s clear now what happens to \( d(h) \) as \( h \to 0 \), it tends to 4 which is the slope of the tangent line at \((2,4)\).
# Note, too that plugging in h=0 into the expression
\[ d(h); \]
\[ \frac{(2 + h)^2 - 4}{h} \]
# seems to yield 0/0 which doesn’t make sense.
# However, by simplifying, figuring out what happens as h \to 0
becomes clear.