Let $S$ be the set of integers. If $a, b \in S$, define $a R b$ if $ab > 0$. Is $R$ an equivalence relation?

[Note: Sometimes the notation $a R b$ is used instead of $a R b$, but the concept is the same.]

The three conditions for an equivalence relation:
1. $a R a$ (reflexivity)
2. $a R b \rightarrow b R a$ (symmetry)
3. $a R b$ and $b R c \rightarrow a R c$ (transitivity)

1. $a R a$?
   - Since $a \in S = \mathbb{Z}$, then $a \cdot a = a^2 > 0$ certainly holds.

2. $a R b \rightarrow ab > 0 \rightarrow b \cdot a > 0 \rightarrow b R a$

3. $a R b \rightarrow ab > 0$ and $b R c \rightarrow bc > 0$
   - So the question is, does this imply that $a R c$ i.e. $ac > 0$?

The answer to this is no. The way to see this is to look for choices of $a, b, c$ such that would give $a R b$, $b R c$ but not $a R c$.

Ex: If $b > 0$ then $ab > 0$ and $bc > 0$ hold for any $a, c$
   - So can we 'cook up' numbers $a, c$ so that $ac < 0$?
     - Yes, just let, for example $a = 2$, $c = -3$

   If so then $ab > 0$, $bc > 0$ but $ac = -3 < 0$
   - (i.e. $ac > 0$ fails to hold)

So $R$ is not an equivalence relation.