MA 411 Homework 3

I. Book Problems

(1) Kaplan p100 1,4,7,11
(2) Kaplan p104-105 1a,2b,4,5

II. Additional Problems.

Consider the function

\[
f(x, y) = \begin{cases} 
\frac{x^2y}{x^2+y^2} & (x, y) \neq (0, 0), \\
0 & (x, y) = (0, 0).
\end{cases}
\]

(1) Use the limit definition of the derivative to compute the directional derivative in the directions (1, 0), (0, 1) and (1, 1).

(2) Compute the dot product \( \nabla f(0, 0) \cdot (1, 1) \).

(3) What can you conclude about the total differential \( df(0,0) \)?

(4) Let \( g(u, v) = (uv, u - v) \), and let \( h(u, v) = f(g(u, v)) \). Write \( dh(1, 0) \) as a product of matrices.

(5) How would you determine whether or not \( dh(0,0) \) exists?

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