MA 511
Problems for Week #1

Your work on these problems will be collected on Wednesday, September 10, in class.

Part A. (September 3, 2003)

1. (Solow) Which of the following are statements?
   a. \( ax^2 + bx + c = 0 \);
   b. \( ax^2 + bx + c \);
   c. \( \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \);
   d. \( \sin \frac{\pi}{2} < \sin \frac{\pi}{4} \);
   e. \( \sin^2 t + \cos^2 t = 1 \);
   f. \( x \in X \).

2. (Solow) Each of the following statements can be expressed as an implication. In each
case, write down the hypothesis and the conclusion. Your answers will be statements.
   a. If the right triangle \( \triangle XYZ \) with legs of lengths \( x \) and \( y \), and hypotenuse of length \( z \), has an area of \( \frac{z^2}{4} \) then \( \triangle XYZ \) is isosceles.
   b. \( n \) is an even integer \( \Rightarrow \) \( n^2 \) is an even integer.
   c. If \( a, b, c, d, e, f \) are real numbers with the property that \( ad - bc \neq 0 \), then the
two linear equations \( ax + by = e \) and \( cx + dy = f \) can be solved for \( x \) and \( y \).
   d. The sum of the squares of the first \( n \) positive integers is \( \frac{(2n+1)n(n+1)}{6} \).
   e. \( r \) is real and satisfies \( r^2 = 2 \) implies \( r \) is irrational.
   f. The value of \( x(x - 1) \) is at least \( -\frac{1}{4} \) for real numbers \( x \).


Part B. (September 5, 2003)

1. (Solow) Consider the problem of proving that “The maximum of \( -x^2 + 2x + 1 \) is \( \geq 2 \)
   for \( x \) real.” Which of the following abstraction questions is ‘incorrect’ and why?
   a. How can I show that the maximum value of a parabola is \( \geq \) to a number?
   b. How can I show that a number is \( \leq \) to the maximum of a polynomial?
   c. How can I show that the maximum value of the function \( -x^2 + 2x + 1 \) is \( \geq \) to a
   number?
   d. How can I show that a number is \( \leq \) to the maximum of a quadratic polynomial?

2. (Solow) For each of the following problems, list as many abstraction questions as you can (at least two). Be sure that your questions contain no symbols or notation from
the specific problem.
   a. If \( \ell_1 \) and \( \ell_2 \) are the tangent lines to a circle \( C \) at the two endpoints \( e_1 \) and \( e_2 \) of
   a diameter \( d \) respectively, then \( \ell_1 \) and \( \ell_2 \) are parallel.
   b. If \( f \) and \( g \) are continuous functions then the function \( f + g \) is continuous.
   c. If \( n \) is an even integer then \( n^2 \) is an even integer.

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3. (Solow) For each of the following abstraction questions, list as many answers as you can (at least three).
   a. How can I show that two real numbers are equal?
   b. How can I show that two triangles are congruent?
   c. How can I show that two lines are parallel?
   d. How can I show that a quadrilateral is a rectangle?