The facts that $|9| = 9$ and $|-9| = 9 = -(-9)$ suggest the following algebraic definition of absolute value.

**Absolute Value**

For any real number $a$,

$$|a| = a \quad \text{if } a \geq 0$$

$$|a| = -a \quad \text{if } a < 0.$$

The first part of the definition shows that $|0| = 0$ (because $0 \geq 0$). It also shows that the absolute value of any positive number $a$ is the number itself, so $|a|$ is positive in such cases. The second part of the definition says that the absolute value of a negative number $a$ is the negative of $a$. For instance, if $a = -5$, then $|-5| = -(-5) = 5$. So $|-5|$ is positive. The same thing works for any negative number—that is, its absolute value (the negative of a negative number) is positive. Thus, we can state the following:

*For every nonzero real number $a$, the number $|a|$ is positive.*

**Example 14** Evaluate $|8 - 9|$.

**Solution** First, simplify the expression within the absolute-value bars:

$$|8 - 9| = |-1| = 1.$$
1.1 The Real Numbers

Business  The nominal annual percentage rate (APR) reported by lenders has the formula \( \text{APR} = 12r \), where \( r \) is the monthly interest rate. Find the APR when

17. \( r = 3.8 \) \( \frac{456}{45} \)
18. \( r = 0.8 \) \( \frac{96}{96} \)

Find the monthly interest rate \( r \) when

19. \( \text{APR} = 11 \) \( \frac{11}{11} \)
20. \( \text{APR} = 13.2 \) \( \frac{13.2}{13.2} \)

Evaluate each expression, using the order of operations given in the text. (See Examples 7–9.)

21. \( 3 - 4 \cdot 5 + 5 - 12 \)
22. \( 8 - (-4)^2 - (-12) \)
23. \( (4 - 5) \cdot 6 + 6 \)
24. \( \frac{2(3 - 7) + 4(8)}{4(-3) + (-3)(-2)} \)
25. \( 8 - 4^2 - (-12) \)
26. \( -(3 - 5) - [2 - (3^2 - 13)] \)
27. \( \frac{2(-3) + 3/(-2) - 2/(\sqrt{16})}{\sqrt{64} - 1} \)
28. \( \frac{6^2 - 3\sqrt{25}}{\sqrt{6^2} + 13} \)

Use a calculator to help you list the given numbers in order from smallest to largest. (See Example 10.)

29. \( \frac{189}{37}, 4587, \sqrt{47}, 6.735, \sqrt{27}, \frac{2040}{523} \)
30. \( \frac{385}{117}, \sqrt{10}, \frac{187}{63}, \pi, \sqrt{85}, 2.9884 \)

Express each of the following statements in symbols, using \(<, >, \leq, \geq, \), or \( \neq \).

31. \( 12 \) is less than \( 18.5 \). \( 12 < 18.5 \)
32. \( -2 \) is greater than \( -20 \). \( -2 > -20 \)
33. \( x \) is greater than or equal to \( 5.7 \). \( x \geq 5.7 \)
34. \( y \) is less than or equal to \( -5 \). \( y \leq -5 \)
35. \( z \) is at most \( 7.5 \). \( z \leq 7.5 \)
36. \( w \) is negative. \( w < 0 \)

Fill in the blank with \(<, =, \text{or } > \) so that the resulting statement is true.

37. \( -6 \text{_____} -2 \text{<} \)
38. \( \frac{3}{4} \text{_____} 0.75 \text{=} \)
39. \( 3.14 \text{_____} \pi \text{<} \)
40. \( 1/3 \text{_____} 0.33 \text{>} \)

Fill in the blank so as to produce two equivalent statements. For example, the arithmetic statement "\( a \) is negative" is equivalent to the geometric statement "the point \( a \) lies to the left of the point \( 0 \)."

**Arithmetic Statement**  **Geometric Statement**

41. \( a \geq b \)  \( a \) lies to the right of \( b \) or is equal to \( b \).
42. \( b + c = a \)  \( b + c \) units to the right of \( b \\
43. \( c < a \)  \( a \) lies between \( b \) and \( c \), and to the right of \( c \\
44. \( a \) is positive  \( a \) lies to the right of \( 0 \).

---

Graph the given intervals on a number line. (See Examples 12 and 13.)

45. \( (-8, -1) \)
46. \( [-1, 10] \)
47. \( (-\infty, -2) \)
48. \( [-2, 2] \)
49. \( (-\infty, -2] \)
50. \( [-\infty, -2) \)

Evaluate each of the following expressions (see Example 14).

51. \( -9 - (-12) \)
52. \( |-8| - |-4| \)
53. \( -|4| - |-1 - 14| \)
54. \( -|6| - |-12 - 4| \)

In each of the following problems, fill in the blank with either \( =, <, \text{or } > \), so that the resulting statement is true.

55. \( |5| \text{_____} |5| \)
56. \( -|-4| \text{_____} |-5| \)
57. \( 10 - 3 \text{_____} 3 - 10 \)
58. \( 6 - (-4) \text{_____} |4 - 6| \)
59. \( |-2 + 8| \text{_____} 2 - 8 \)
60. \( |3| - |-5| \text{_____} |3 - 5| \)
61. \( 3 - 5 \text{_____} |3 - 5| \)
62. \( |-5 + 1| \text{_____} |5| + |1| \)

Write the expression without using absolute-value notation.

63. \( |a - 7| \text{if } a < 7 \text{<} \)
64. \( |b - c| \text{if } b \geq c \)

65. If \( a \) and \( b \) are any real numbers, is it always true that \( |a + b| = |a| + |b| \) ? Explain your answer.
66. If \( a \) and \( b \) are any two real numbers, is it always true that \( |a - b| = |b - a| \) ? Explain your answer.
67. For which real numbers \( b \) does \( |2 - b| = |2 + b| \) ? Explain your answer.

68. Health  Data from the National Health and Nutrition Examination Study estimates that 95% of adult heights (inches) are in the following ranges for females and males. (Data from: www.cdc.gov/nchs/nhanes.htm.)

<table>
<thead>
<tr>
<th></th>
<th>Females</th>
<th>Males</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>63.5 ± 8.4</td>
<td>68.9 ± 9.3</td>
</tr>
</tbody>
</table>

Express the ranges as an absolute-value inequality in which \( x \) is the height of the person.

69. \( |x - 63.5| \leq 8.4; |x - 68.9| \leq 9.3 \)

Business  The Consumer Price Index (CPI) tracks the cost of a typical sample of consumer goods. The following table shows the percentage increase in the CPI for each year in a 10-year period.

<table>
<thead>
<tr>
<th>Year</th>
<th>2003</th>
<th>2004</th>
<th>2005</th>
<th>2006</th>
<th>2007</th>
</tr>
</thead>
<tbody>
<tr>
<td>% Increase in CPI</td>
<td>2.3</td>
<td>2.7</td>
<td>2.5</td>
<td>3.2</td>
<td>4.1</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Year</th>
<th>2008</th>
<th>2009</th>
<th>2010</th>
<th>2011</th>
<th>2012</th>
</tr>
</thead>
<tbody>
<tr>
<td>% Increase in CPI</td>
<td>0.1</td>
<td>2.7</td>
<td>1.5</td>
<td>3.0</td>
<td>1.7</td>
</tr>
</tbody>
</table>

---

Footnote: Indicates answer is in the Additional Instructor Answers at end of the book.
Checkpoint 10

Suppose revenue is given by \(7x^2 - 3x\), fixed costs are $500, and variable costs are given by \(3x^3 + 5x - 25\). Write an expression for:

(a) Cost
(b) Profit

where \(x \leq 150\) (because only 150,000 calculators can be made). The variable cost of making \(x\) thousand calculators is 6995x - 7.2x^2, so that

\[
\text{Cost} = \text{Fixed Costs} + \text{Variable Costs},
\]

\[
C = 230,000 + (6995x - 7.2x^2) \quad (x \leq 150).
\]

Therefore, the profit is given by

\[
P = R - C = 12,000x - (230,000 + 6995x - 7.2x^2)
\]

\[
= 12,000x - 230,000 - 6995x + 7.2x^2
\]

\[
P = 7.2x^2 + 5005x - 230,000 \quad (x \leq 150).
\]

1.2 Exercises

Use a calculator to approximate these numbers. (See Examples 1 and 2.)

1. \(11.2^6 \approx 1,973,422.685\)
2. \((-6.54)^{11} \approx -936,171,103.1\)
3. \((-18.77)^6 \approx 389,699,139\)
4. \((5.9)^7 \approx 910,633,996.7\)

5. Explain how the value of \((-3)^3\) differs from \((-3)^2\). Do \(-3^3\) and \((-3)^3\) differ in the same way? Why or why not? Answer: vary.

6. Describe the steps used to multiply \(4^3\) and \(4^2\). Is the product of \(4^3\) and \(4^2\) found in the same way? Explain. Answer: vary.

Simplify each of the given expressions. Leave your answers in exponential notation. (See Examples 3 and 4.)

7. \(4^2 \cdot 4^3 = 4^{2+3} = 4^5\)
8. \((-4)^4 \cdot (-4)^6 = (-4)^{4+6} = (-4)^{10}\)
9. \((-6)^3 \cdot (-6)^5 = (-6)^{3+5} = (-6)^8\)
10. \((2x)^3 \cdot (2x)^6 = (2x)^{3+6} = (2x)^9\)
11. \([5(6^3)]^4 = (6^3)^4 = [6^{3\cdot4}] = (6^{12})^{1/4}\)

List the degree of the given polynomial, its coefficients, and its constant term. (See Example 5.)

13. \(6x^4 - 5x^3 + 4x^2 - 3x + 3.7\) Degree: 4, coefficients: 6, -5, 4, -3, 3.7; constant term 3.7
14. \(6x^3 + 4x^2 - x^3 + x\) Degree: 3, coefficients: 6, 4, -1, 0, 1, 0; constant term 0

State the degree of the given polynomial.

15. \(1 + x + 2x^2 + 3x^3\) Degree: 3
16. \(5x^4 - 4x^3 - 6x^2 + 7x^4 - 2x + 8\) Degree: 4

Add or subtract as indicated. (See Examples 6 and 7.)

17. \((3x^3 + 2x^2 - 5x) + (-4x^3 - 3x^2 - 8x) = -x^3 - x^2 - 13x\)
18. \((-2x^3 - 5p + 7) + (-4p^2 + 8p + 2) = -2x^3 - 4p^2 + 3y + 9\)
19. \((-4y^3 - 3y + 8) - (2y^3 - 6y + 2) = -6y^3 + 3y + 6\)
20. \((7b^2 + 2b - 5) - (3b^2 + 2b - 6) = 4b^2 + 1\)
21. \((2x^3 + 2x^2 + 4x - 3) - (2x^3 + 8x^2 + 1) = -6x^2 + 4x - 4\)
22. \((3y^3 + 9y^2 - 11y + 8) - (-4y^3 + 10y - 6)\)

Find each of the given products. (See Examples 8–10.)

23. \(-9m(2m^2 + 6m - 1) = -18m^3 - 54m^2 + 9m\)
24. \(2a(4a^2 - 6a + 8) = 8a^3 - 12a^2 + 16a\)

25. \((3z + 5)(4z^2 - 2z + 1) = 12z^3 + 16z^2 - 7z + 5\)
26. \((2k + 3)(4k^3 - 3k^2 + k) = 8k^4 + 6k^3 - 3k^2 + 3k\)
27. \((6k - 1)(2k + 3) = 12k^2 + 16k - 3\)
28. \((8r + 3)(r - 1) = 8r^2 - 5r - 3\)
29. \((3y + 5)(2y + 1) = 6y^2 + 13y + 5\)
30. \((5r - 3)(5r - 4a) = 25r^2 - 35ax + 19r\)
31. \((9k + q)(2k - q) = 18k^2 - 7qk - q^2\)
32. \((.012x - .17)(3x + .54) = .0436x^2 - .0452x - .0918\)
33. \((6.2m^3 - 3.4)(7m + 1.3) = 4.34m^4 + 5.60am - 4.42\)
34. \(2p - 3[4p - (8p + 1)] = 16p + 3\)
35. \(5k - [k + (-3 + 5k)] = -k + 3\)
36. \((3x - 1)(x + 2) - (2x + 5)^2 = -x^2 - 15x - 23\)

Business: Find expressions for the revenue, cost, and profit from selling x thousand items. (See Example 11.)

<table>
<thead>
<tr>
<th>Item Price</th>
<th>Fixed Costs</th>
<th>Variable Costs</th>
</tr>
</thead>
<tbody>
<tr>
<td>$7.50</td>
<td>$200,000</td>
<td>$1800x</td>
</tr>
<tr>
<td>$8.50</td>
<td>$225,000</td>
<td>$2400x</td>
</tr>
</tbody>
</table>

Business: Beauty Works sells its cologne wholesale for $9.75 per bottle. The variable costs of producing x thousand bottles is \(-3x^2 + 3480x - 325\) dollars, and the fixed costs of manufacturing are $260,000. Find expressions for the revenue, cost, and profit from selling x thousand items.

Business: A self-help guru sells her book Be Happy in 45 Easy Steps for $23.50 per copy. Her fixed costs are $145,000 and she estimates the variable cost of printing, binding, and distribution is given by \(-4.2x^2 + 3220x - 425\) dollars. Find expressions for the revenue, cost, and profit from selling x thousand copies of the book.

Work these problems.

Business: The accompanying bar graph shows the net earnings (in millions of dollars) of the Starbucks Corporation. The polynomial

\[-1.48x^4 + 50.0x^3 - 576x^2 + 2731x - 4027\]
gives a good approximation of Starbucks’s net earnings in year
(where x = 3 corresponds to 2003, x = 4 to 2004, and so on
(3 ≤ x ≤ 12). For each of the given years,
(a) use the bar graph to determine the net earnings;
(b) use the polynomial to determine the net earnings.
(Data from: www.morningstar.com.)

<table>
<thead>
<tr>
<th>Year</th>
<th>2003</th>
<th>2007</th>
<th>2010</th>
<th>2012</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$3,200</td>
<td>$3,200</td>
<td>$3,200</td>
<td>$3,200</td>
</tr>
<tr>
<td></td>
<td>(a) $2,570</td>
<td>(a) $2,570</td>
<td>(a) $2,570</td>
<td>(a) $2,570</td>
</tr>
<tr>
<td></td>
<td>(b) $650</td>
<td>(b) $650</td>
<td>(b) $650</td>
<td>(b) $650</td>
</tr>
<tr>
<td></td>
<td>(c) $1,650</td>
<td>(c) $1,650</td>
<td>(c) $1,650</td>
<td>(c) $1,650</td>
</tr>
<tr>
<td></td>
<td>(d) $420</td>
<td>(d) $420</td>
<td>(d) $420</td>
<td>(d) $420</td>
</tr>
<tr>
<td></td>
<td>(e) $1,200</td>
<td>(e) $1,200</td>
<td>(e) $1,200</td>
<td>(e) $1,200</td>
</tr>
</tbody>
</table>

Assuming that the polynomial approximation in Exercises 41–44
remains accurate in later years, use it to estimate Starbucks’s net
earnings in each of the following years.

45. 2013 (about $1,712 million) 46. 2014 (about $1,655 million) 47. 2015 (about $1,613 million)


Economics The percentage of persons living below the poverty
line in the United States in year x is approximated by the polynomial
\[-0.0057x^4 + 0.157x^3 - 1.43x^2 + 5.14x + 6.3, \text{ where } x = 0\]
corresponds to the year 2000. Determine whether each of the given
statements is true or false. (Data from: U.S. Census Bureau, Current
Population Survey, Annual Social and Economic Supplements.)

49. The percentage living in poverty was higher than 13% in
2004. True

50. The percentage living in poverty was higher than 14% in 2010. True

51. The percentage living in poverty was higher in 2003 than
2006. False

52. The percentage living in poverty was lower in 2009 than
2008. True

Health According to data from a leading insurance company, if
a person is 65 years old, the probability that he or she will live for
another x years is approximated by the polynomial
\[1 - 0.0058x - 0.0076x^2.\]
(Data from: Ralph DeMarr, University of New Mexico.)

Find the probability that a 65-year-old person will live to the
following ages.

53. 75 (that is, 10 years past 65) 0.666 54. 80 0.742
55. 87 0.205 56. 95 0.142
57. Physical Science One of the most amazing formulas in all
of ancient mathematics is the formula discovered by the
Egyptians to find the volume of the frustum of a square pyra-
mid, as shown in the following figure:

\[
\frac{1}{3}h \cdot (a^2 + ab + b^2),
\]
where b is the length of the base, a is the length of the top, and

(a) When the Great Pyramid in Egypt was partially completed
to a height h of 200 feet, b was 756 feet and a was 314 feet.
Calculate its volume at this stage of construction.
(Approximately 91,600,000 cubic feet)

(b) Try to visualize the figure if a = b. What is the resulting
shape? Find its volume. The shape becomes a rectangular box
with a square base, with volume 6h^2.

(c) Let a = b in the Egyptian formula and simplify. Are the
results the same? Yes

58. Physical Science Refer to the formula and the discussion in
Exercise 57.

(a) Use the expression \((1/3)h(a^2 + ab + b^2)\) to determine a
formula for the volume of a pyramid with a square base b
and height h by letting a = 0.

(b) The Great Pyramid in Egypt had a square base of length
756 feet and a height of 481 feet. Find the volume of the Great
Pyramid. Compare it with the volume of the 273-foot-tall
Louisiana Superdome, which has an approximate volume
of 125 million cubic feet. (Data from: Louisiana Super-
dome [www.superdome.com].) About 91.6 million cubic feet,
slightly smaller than the Superdome.

(c) The Superdome covers an area of 13 acres. How many
acres does the Great Pyramid cover? (Hint: 1 acre =
43,560 ft^2). 13.1 acres

59. Suppose one polynomial has degree 3 and another also has
degree 3. Find all possible values for the degree of their
(a) sum; 0, 1, 2, 3, or no degree (if one is the negative of the other)
(b) difference; 0, 1, 2, 3, or no degree (if they are equal)
(c) product; 6

Business Use the table feature of a graphing calculator or use
a spreadsheet to make a table of values for the profit function in
Example 11, with x = 0, 5, 10, . . . , 150. Use the table to answer
the following questions.

60. What is the profit or loss (negative profit) when 25,000 calcu-
ulators are sold? When 60,000 are sold? Explain these
answers. $100,375, $95,720

61. Approximately how many calculators must be sold in order for
the company to make a profit? Between 40,000 and 45,000 calculators

62. What is the profit from selling 100,000 calculators? 150,000
calculators? $492,900, $589,740
Checkpoint 8
Factor the following.
(a) $a^3 + 1000$
(b) $x^2 - 64$
(c) $1000m^3 - 27z^3$

(b) $m^3 + 125$
Solution $m^3 + 125 = m^3 + 5^3 = (m + 5)(m^2 - 5m + 25)$

(c) $8k^3 - 27z^3$
Solution $8k^3 - 27z^3 = (2k)^3 - (3z)^3 = (2k - 3z)(4k^2 + 6kz + 9z^2)$

Substitution and appropriate factoring patterns can sometimes be used to factor higher degree expressions.

Example 11  Factor the following polynomials.
(a) $x^8 + 4x^4 + 3$
Solution  The idea is to make a substitution that reduces the polynomial to a quadratic or cubic that we can deal with. Note that $x^8 = (x^4)^2$. Let $u = x^4$. Then

$$x^8 + 4x^4 + 3 = (x^4)^2 + 4x^4 + 3$$

Power of a power

$$= u^2 + 4u + 3$$

Substitute $x^4 = u$.

$$= (u + 3)(u + 1)$$

Factor.

$$= (x^4 + 3)(x^4 + 1)$$

Substitute $u = x^4$.

(b) $x^4 - y^4$
Solution  Note that $x^4 = (x^2)^2$, and similarly for the $y$ term. Let $u = x^2$ and $v = y^2$. Then

$$x^4 - y^4 = (x^2)^2 - (y^2)^2$$

Power of a power

$$= u^2 - v^2$$

Substitute $x^2 = u$ and $y^2 = v$.

$$= (u + v)(u - v)$$

Difference of squares.

$$= (x^2 + y^2)(x^2 - y^2)$$

Substitute $u = x^2 + y = y^2$.

$$= (x^2 + y^2)(x + y)(x - y).$$

Difference of squares.

Once you understand Example 11, you can often factor without making explicit substitutions.

Example 12  Factor $256k^4 - 625m^4$.
Solution  Use the difference of squares twice, as follows:

$$256k^4 - 625m^4 = (16k^2)^2 - (25m^2)^2$$

$$= (16k^2 + 25m^2)(16k^2 - 25m^2)$$

$$= (16k^2 + 25m^2)(4k + 5m)(4k - 5m).$$

Checkpoint 9
Factor each of the following.
(a) $2x^4 + 5x^2 + 2$
(b) $3x^4 - x^2 - 2$

Checkpoint 10
Factor $81x^4 - 16y^4$.

1.3 Exercises

Factor out the greatest common factor in each of the given polynomials. (See Example 1.)

1. $12x^2 - 24x - 12x(x - 2)$
2. $5y - 65xy - 5y(1 - 13x)$
3. $3x^3 - 3x^2 + x(x^2 - 1) + 13x$  
4. $x^2 + 3y^2 + 8 + 6x^2 + 3x + 8$  
5. $x^2 - 12x + 18$  
6. $5x^3 + 55x^2 + 10x$  
7. $3(2y - 1)^2 + 7(2y - 1)^3$  
8. $(3x + 7)^3 - 4(3x + 7)^2$  
9. $(x + 5)^4 + (x + 5)^6$  
10. $(x + 6)^2 + 6(x + 6)^4$  

11. $(x + 5x + 4)$  
12. $r^2 + 7u + 6$  
13. $x^2 + 7x + 12$  
14. $y^2 + 8y + 12$  
15. $x^2 + x - 6$  
16. $x^2 + 4x - 5$  
17. $x^2 + 2x - 3$  
18. $y^2 + y - 12$  
19. $x^2 - 3x - 4$  
20. $x^2 - 2u - 8$  

Factor the polynomial. (See Examples 2 and 3.)
21. \( z^2 - 9z + 14 \)
   
22. \( w^2 - 6w + 16 \)

23. \( z^2 + 10z + 24 \)

24. \( r^2 + 16r + 60 \)

Factor the polynomial. (See Examples 4–6.)

25. \( 2x^2 - 9x + 4 \)

26. \( 3w^2 - 8w + 4 \)

27. \( 15p^2 - 23p + 4 \)

28. \( 8x^2 - 14x + 3 \)

29. \( 4c^2 - 16c + 15 \)

30. \( 12y^2 - 29y + 15 \)

31. \( 6x^2 - 5x - 4 \)

32. \( 12x^2 + 2x - 1 \)

33. \( 10y^2 + 21y - 10 \)

34. \( 15u^2 + 4u - 4 \)

35. \( 6x^2 + 5x - 4 \)

36. \( 12y^2 + 7y - 10 \)

Factor each polynomial completely. Factor out the greatest common factor as necessary. (See Examples 2–9.)

37. \( 3a^2 + 2a - 5 \)

38. \( 6x^2 - 48x - 120 \)

39. \( x^2 - 81 \)

40. \( x^2 + 17xy + 72y^2 \)

41. \( 9p^2 - 12p + 4 \)

42. \( 3p^2 - r + 2 \)

43. \( x^2 + 3xr + 10x^2 \)

44. \( 2a^2 + ab - 6b^2 \)

45. \( m^2 - 8mn + 16n^2 \)

46. \( 8x^2 - 16x - 10 \)

47. \( 4u^2 + 12u + 9 \)

48. \( 9y^2 - 18y + 9 \)

49. \( 25p^2 - 10p + 4 \)

50. \( 4x^2 - 10x + 3 \)

51. \( 4r^2 - 9s^2 \)

52. \( x^2 + 3xy - 28x^2 \)

53. \( x^2 + 4xy + 4y^2 \)

54. \( 15x^2 + 12x - 18 \)

55. \( 3x^2 - 13x - 30 \)

56. \( 36x^2 + 56xy + 196y^2 \)

57. \( 21m^2 + 13mn + 2n^2 \)

58. \( 81y^2 - 100 \)

59. \( y^2 - 4yz - 21z^2 \)

60. \( 121x^2 - 64 \)

61. \( 12x^2 - 64 \)

Factor each of these polynomials. (See Example 10.)

62. \( a^2 + 64 \)

63. \( 64 \)

64. \( b^2 + 216 \)

65. \( 8x^2 - 27x^3 \)

66. \( 1000y^2 + 27y^2 \)

67. \( 64m^2 + 125 \)

68. \( 216y^3 - 343 \)

69. \( 1000y^2 - z^2 \)

Factor each of these polynomials. (See Examples 11 and 12.)

70. \( x^4 + 5x^2 + 6 \)

71. \( x^4 + 7x^2 + 10 \)

72. \( y^4 + 7y^2 + 10 \)

73. \( 9p^3 + 19p^2 - 1 \)

74. \( c^3 - 3c^2 - 4 \)

75. \( x^4 - x^2 - 12 \)

76. \( 4x^4 + 27x^2 - 81 \)

77. \( 16a^4 - 81b^6 \)

78. \( x^8 - y^8 \)

79. \( x^8 + 8x^2 \)

80. \( x^9 - 64x^3 \)

81. When asked to factor \( 6x^4 - 3x^2 - 3 \) completely, a student gave the following result:

\[ 6x^4 - 3x^2 - 3 = (2x^2 + 1)(3x^2 - 3). \]

Is this answer correct? Explain why. Answer: no.

82. When can the sum of two squares be factored? Give examples. Answer: never.

83. Explain why \((x + 2)^2\) is not the correct factorization of \(x^2 + 8\), and give the correct factorization. Answer: \((x + 2)(x - 2)\).

84. Describe how factoring and multiplication are related. Give examples. Answer: they are inverses of each other.

### Checkpoint Answers

1. (a) \(3(4r + 3k)\)

   (b) \(25(3n^2 + 4n^2)\)

2. (a) \((r + 2)(r + 5)\)

   (b) \((x + 3)(x + 1)\)

3. (a) \((x - 3)(x - 1)\)

   (b) \((2y - 1)(y + 2)\)

4. (a) \((r - 7)(r + 2)\)

   (b) \((3m - 1)(m + 2)\)

5. (a) \((3p + 7)(3p - 7)\)

   (b) Cannot be factored

6. (a) \((2m + 1)^2\)

   (b) \((5z - 8)^2\)

7. (a) \((3x + 1)(x - 5)\)

   (b) \((3r + 2 - n)(3r + 2 + n)\)

8. (a) \((a + 10)(a^2 - 10a + 100)\)

   (b) \((z - 4)(z^2 + 4z + 16)\)

   (c) \((10m - 3z)(100m^2 + 30mz + 9z^2)\)

9. (a) \((2x^2 + 1)(x^2 + 2)\)

   (b) \((3x^2 + 2)(x + 1)(x - 1)\)

10. (a) \((9x^2 + 4y^2)(3x + 2y)(3x - 2y)\)

### 1.4 Rational Expressions

A rational expression is an expression that can be written as the quotient of two polynomials, such as

\[ \frac{8}{x - 1}, \quad \frac{3x^2 + 4x}{5x - 6}, \quad \text{and} \quad \frac{2y + 1}{y^4 + 8} \]
\[
\frac{k^2}{(k + 1)(k - 1)} - \frac{2k^2 - k - 3}{(k + 1)(k + 2)} \\
= \frac{k^2(k + 2)}{(k + 1)(k - 1)(k + 2)} - \frac{(2k^2 - k - 3)(k - 1)}{(k + 1)(k - 1)(k + 2)} \\
= \frac{k^3 + 2k^2 - (2k^2 - k - 3)(k - 1)}{(k + 1)(k - 1)(k + 2)} \\
= \frac{k^3 + 2k^2 - 2k^3 - 3k^2 - 2k + 3}{(k + 1)(k - 1)(k + 2)} \\
= \frac{k^3 + 2k^2 - 2k^3 + 3k^2 + 2k - 3}{(k + 1)(k - 1)(k + 2)} \\
= \frac{-k^3 + 5k^2 + 2k - 3}{(k + 1)(k - 1)(k + 2)}.
\]

### Complex Fractions

Any quotient of rational expressions is called a **complex fraction**. Complex fractions are simplified as demonstrated in Example 6.

**Example 6** Simplify the complex fraction

\[
\frac{6 - \frac{5}{k}}{1 + \frac{5}{k}}
\]

**Solution** Multiply both numerator and denominator by the common denominator \(k\):

\[
\frac{6 - \frac{5}{k}}{1 + \frac{5}{k}} = \frac{k \left( 6 - \frac{5}{k} \right)}{k \left( 1 + \frac{5}{k} \right)}
\]

Multiply by \(\frac{k}{k}\):

\[
6k - k \left( \frac{5}{k} \right)
\]

Distributive property:

\[
= \frac{6k - 5}{k + 5}.
\]

Simplify.

### 1.4 Exercises

Write each of the given expressions in lowest terms. Factor as necessary. (See Example 1.)

1. \(\frac{8x^2}{56x} \frac{x}{7}\)
2. \(\frac{27m}{81m^2} \frac{1}{2m^2}\)
3. \(\frac{25p^2}{35p^3} \frac{5}{3p}\)
4. \(\frac{18y^4}{24y^2} \frac{3y^2}{4}\)
5. \(\frac{5m + 15}{4m + 12} \frac{5}{4}\)
6. \(\frac{10z + 5}{20z + 10} \frac{1}{2}\)
7. \(\frac{4(w - 3)}{(w - 3)(w + 6)} \frac{4}{w + 6}\)
8. \(\frac{-6(x + 2)}{(x + 4)(x + 2)} \frac{6}{x + 4}\)
9. \(\frac{3y^2 - 12y}{9y^3} \frac{y - 4}{3y^2}\)
10. \(\frac{15k^2 + 45k}{9k^2} \frac{5(k + 3)}{3k}\)
11. \(\frac{m^2 - 4m + 4}{m^2 + m - 6} \frac{m - 2}{m + 3}\)
12. \(\frac{r^2 - r - 6}{r^3 + r - 12} \frac{r + 2}{r + 4}\)
13. \(\frac{x^2 + 2x - 3}{x^2 - 1} \frac{x + 3}{x - 1}\)
14. \(\frac{z^2 + 4z + 4}{z^2 - 4} \frac{z + 2}{z - 2}\)
Multiply or divide as indicated in each of the exercises. Write all answers in lowest terms. (See Examples 2 and 3.)

15. \( \frac{3a^2}{64} \cdot \frac{8}{2a^2} \)
16. \( \frac{2r^2}{3} \cdot \frac{10u^3}{9u} \)
17. \( \frac{7x}{11} + \frac{14x^2}{66y} \)
18. \( \frac{6x^2y}{2x} + \frac{21xy}{y} \)
19. \( \frac{2a + b}{3c} \cdot \frac{15}{4(2a + b)} \)
20. \( \frac{3x^2}{w} \cdot \frac{4(x + 2)}{8(x + 2)} \)
21. \( \frac{15p - 3}{6} \cdot \frac{10p - 2}{3} \)
22. \( \frac{2k + 8}{6} + \frac{3k + 12}{3} \)
23. \( \frac{9y - 18}{3y + 6} \cdot \frac{6y + 12}{15y - 30} \)
24. \( \frac{12r + 24}{36r - 36} + \frac{6r + 12}{8r - 8} \)
25. \( \frac{4a + 12}{2a - 10} + \frac{a^2 - 9}{a^2 - a - 20} \)
26. \( \frac{6r - 18}{9r^2 + 6r - 24} + \frac{12r - 16}{4r - 12} \)
27. \( \frac{k^2 - k - 6}{k^2 + k - 12} \cdot \frac{k^2 + 3k - 4}{k^2 - 2k - 3} \)
28. \( \frac{n^2 - n - 6}{n^2 - 2n - 8} \div \frac{n^2 - 9}{n^2 + 7n + 12} \)

29. In your own words, explain how to find the least common denominator of two fractions. Answers vary.
30. Describe the steps required to add three rational expressions. You may use an example to illustrate. Answers vary.

Add or subtract as indicated in each of the following. Write all answers in lowest terms. (See Example 4.)

31. \( \frac{2}{7z} - \frac{1}{5z} \)
32. \( \frac{4}{3z} - \frac{5}{4z} - \frac{1}{12z} \)
33. \( \frac{r + 2}{3} - \frac{r - 2}{3} \)
34. \( \frac{3y - 1}{8} - \frac{3y + 1}{8} - \frac{1}{4} \)
35. \( \frac{4}{x} + \frac{1}{5} \cdot \frac{26 + x}{x} \)
36. \( \frac{6}{r} - \frac{3}{4} \cdot \frac{3r - 2}{3} \)
37. \( \frac{1}{m - 1} + \frac{2}{m} \cdot \frac{3m - 2}{3(m - 1)} \)
38. \( \frac{8}{y + 2} - \frac{3}{y} \cdot \frac{5y - 6}{3(y + 2)} \)
39. \( \frac{7}{b + 2} + \frac{2}{5(b + 2)} \)
40. \( \frac{4}{3(k + 1)} + \frac{3}{k + 1} \)
41. \( \frac{2}{5(k - 2)} + \frac{5}{4(k - 2)} \cdot \frac{20(k - 2)}{20(k - 2)} \)
42. \( \frac{11}{3(p + 4)} - \frac{5}{6(p + 4)} \)
43. \( \frac{2}{x^2 - 4x + 3} + \frac{5}{x^2 - x - 6} \cdot \frac{7x - 1}{(x - 3)(x + 1)(x + 2)} \)
44. \( \frac{3}{m^2 - 3m - 10} + \frac{7}{m^2 - m - 20} \cdot \frac{10m + 26}{(m - 3)(m + 2)(m + 4)} \)

45. \( \frac{2y}{y^2 + 7y + 12} - \frac{y}{y^2 + 3y + 6} \cdot \frac{y}{y + 4(3y + 3)(y + 2)} \)
46. \( \frac{-r}{r^2 - 10r + 16} - \frac{3r}{r^2 + 2r - 8} \cdot \frac{-4r^2 + 20r}{(r - 4)(3)(r + 4)} \)

In each of the exercises in the next set, simplify the complex fraction. (See Example 6.)

47. \( \frac{1 + \frac{1}{x}}{x + 1} \)
48. \( \frac{2}{y - 1} \cdot \frac{1}{y + 1} \)
49. \( \frac{1}{x + h} - \frac{1}{x} \cdot \frac{1}{h} \)
50. \( \frac{(x + h)^2 - x^2}{(x + h)h} \cdot \frac{2x - h}{(x + h)(x - h)} \)

Work these problems.

Natural Science Each figure in the following exercises is a dartboard. The probability that a dart hits the board lands in the shaded area is the fraction

\[
\frac{\text{area of the shaded region}}{\text{area of the dartboard}}
\]

(a) Express the probability as a rational expression in \( x \). (Hint: Area formulas are given in Appendix B.)
(b) Then reduce the expression to lowest terms.

51. ![Diagram of a dartboard]
52. ![Diagram of a dartboard]

53. ![Diagram of a dartboard]
54. ![Diagram of a dartboard]

Business In Example 11 of Section 1.2, we saw that the cost \( C \) of producing \( x \) thousand calculators is given by

\[
C = -7.2x^2 + 6995x + 230,000 \quad (x \leq 150).
\]

55. Write a rational expression that gives the average cost per calculator when \( x \) thousand are produced. (Hint: The average cost is the total cost \( C \) divided by the number of calculators produced.)

\[
\frac{-7.2x^2 + 6995x + 230,000}{1000x} \]

56. Find the average cost per calculator for each of these production levels: 20,000, 50,000, and 125,000. \$18.35, \$11.24, \$7.94
1.5 Exponents and Radicals

Perform the indicated operations and simplify your answer. (See Examples 1 and 2.)

1. \( \frac{7^5}{7^3} = 7^{5-3} = 7^2 = 49 \)

2. \( \frac{(-6)^4}{-6^0} = 6^4 = 1,296 \)

3. \( (6e)^2 = 16e^4 \)

4. \( -2x^4 = 16e^4 \)

5. \( \left( \frac{2}{x} \right)^5 = 32x^{-5} \)

6. \( \left( \frac{5}{xy} \right)^3 = \frac{125}{x^3y^3} \)

7. \( (3p)^2(2a)^2 = 36pa^2 \)

8. \( \frac{(5y)^3}{(2y)^2} = \frac{125y^3}{16} \)

Perform the indicated operations and simplify your answer, which should not have any negative exponents. (See Examples 3 and 4.)

9. \( 7^{-1} = \frac{1}{7} \)

10. \( 10^{-3} = \frac{1}{1000} \)

11. \( -6^{-3} = -\frac{1}{216} \)

12. \( -x^{-4} = \frac{1}{x^4} \)

13. \( (x^{-3})^{-1} = x^3 \)

14. \( \left( \frac{1}{6} \right)^{-2} = 36 \)

15. \( \left( \frac{4}{3} \right)^{-2} = \frac{9}{16} \)

16. \( \left( \frac{x}{y} \right)^{-2} = \frac{y^2}{x^2} \)

17. \( \left( \frac{a}{b} \right)^{-1} = \frac{b}{a} \)

18. Explain why \(-2^{-4} = -\frac{1}{16}\), but \((-2)^{-4} = 1/16\). Answer: vary.

Evaluate each expression. Write all answers without exponents. Round decimal answers to two places. (See Examples 5 and 6.)

19. \( 49^{1/2} = 7 \)

20. \( 8^{1/3} = 2 \)

21. \( (5.71)^{1/4} \approx 1.55 \)

22. \( 125^{1/3} = 5 \)

23. \( -64^{1/3} = -4 \)

24. \( -64^{1/2} = -8 \)

25. \( 8^{2/3} = 16 \)

26. \( 27^{1/3} = 3 \)

Simplify each expression. Write all answers using only positive exponents. (See Example 7.)

27. \( \frac{5^3}{4^2} = 4^2 \cdot 5^3 = 100 \)

28. \( \frac{7^4}{7^3} = 7 \)

29. \( 4^{-3} \cdot 4^5 = 4^2 \)

30. \( 9^{-9} \cdot 9^{10} = 9 \)

31. \( \frac{4^{10} \cdot 4^{-6}}{4^4} = 4^{-2} \)

32. \( \frac{5^{-4} \cdot 5^6}{5^1} = 5^3 \)

Simplify each expression. Assume all variables represent positive real numbers. Write answers with only positive exponents. (See Examples 8 and 9.)

33. \( \frac{x^6 \cdot x^2}{x^5} = x^3 \)

34. \( \frac{k^8 \cdot k^9}{k^{12}} = k^5 \)

35. \( \frac{3^{-1}(p^2 - 2)^3}{3p^7} = \frac{p}{9} \)

36. \( \frac{(5x)^{3/2}}{x^{1/2}} = \frac{1}{25x^0} \)

37. \( (x^{-3}y)^{-1} = \frac{y^3}{x^3} \)

38. \( (2y^2z^{-3})^{-3} = \frac{8y^6}{z^9} \)

39. \( (2p^{-3})^2 = \frac{1}{2p^6} \)

40. \( (3x^{-3})^3 = \frac{27x^{-9}}{x^0} \)

41. \( (2p^{1/2})(2p^{1/3})^{1/3} = 2^{1/2}(2p^{1/3})^{1/3} \)

42. \( (5x)^{3/2} \cdot (5x)^{1/3} = 5^{1/2} \cdot 5^{1/3} \)

43. \( 2^{1/2}(2p^{1/3} + 5p^{1/2}) = 2^{1/2}(5p^{1/3} + 5p^{1/2}) \)

44. \( 3^{1/2}(3x^{1/2} + x^{1/2}) = 3^{1/2}(x^{1/2} + x^{1/2}) \)

45. \( (x^2)^{1/3}(y^2)^{1/3} = \frac{1}{3y^{1/3}} \)

46. \( (x^2)^{1/3}(y^2)^{1/2} = \frac{1}{3x^{1/3}} \)

47. \( (7a)^{3/2}(7b)^{3/2} = 49a^{3/2}b^{3/2} \)

48. \( (4x)^{1/2}(3y)^{1/2} = 2x^{1/2}y^{1/2} \)

49. \( x^{1/2}(x^{1/2} - x^{1/2}) = x^{1/2} \)

50. \( x^{1/2}(x^{1/2} + x^{-1/2}) = x^{1/2} \)

51. \( x^{1/2} = x^{1/2} \)

52. \( x^{1/2} = x^{1/2} \)

53. \( (3x)^{1/3} = 3^{1/3}x^{1/3} \)

54. \( (3x)^{-1/3} = 3^{-1/3}x^{-1/3} \)

55. \( (3x)^{1/3} = 3^{1/3}x^{1/3} \)

56. \( (3x)^{-1/3} = 3^{-1/3}x^{-1/3} \)

57. \( (3x)^{1/3} = 3^{1/3}x^{1/3} \)

58. \( (3x)^{-1/3} = 3^{-1/3}x^{-1/3} \)

59. \( (3x)^{-1/3} = 3^{-1/3}x^{-1/3} \)

60. \( (3x)^{-1/3} = 3^{-1/3}x^{-1/3} \)

Simplify each of the given radical expressions. (See Examples 11–13.)

61. \( \sqrt{125} = 5 \)

62. \( \sqrt{64} = 8 \)

63. \( \sqrt{625} = x \)

64. \( \sqrt{128} = 2 \)

65. \( \sqrt{63} = 7 \)

66. \( \sqrt{81} = 9 \)

67. \( \sqrt{81} = 9 \)

68. \( \sqrt{49} = 7 \)

69. \( \sqrt{64} = 8 \)

70. \( \sqrt{16} = 4 \)

71. \( \sqrt{72} = 6 \)

72. \( \sqrt{75} + \sqrt{192} = 13 \)
73. \(5\sqrt{20} - \sqrt{45} + 2\sqrt{80} \quad \frac{5\sqrt{5}}{2}\) 74. \((\sqrt{3} + 2)(\sqrt{3} - 2)\) 75. \((\frac{1}{3} + \sqrt{3})(\frac{1}{3} - \sqrt{3})\) 76. What is wrong with the statement \(\sqrt{4} \cdot \sqrt{4} = 4?\) Answer: yes.

Rationalize the denominator of each of the given expressions. (See Example 14.)

77. \(\frac{3}{1 - \sqrt{2}} \quad 3 - 3\sqrt{2}\) 78. \(\frac{2}{1 + \sqrt{5}} \quad \frac{\sqrt{5} - 1}{2}\) 79. \(\frac{9 - \sqrt{3}}{3 - \sqrt{3}} \quad 3 + \sqrt{3}\) 80. \(\frac{\sqrt{3} - 1}{\sqrt{3} - 2} \quad 1 - \sqrt{3}\)

Rationalize the numerator of each of the given expressions. (See Example 15.)

81. \(\frac{3 - \sqrt{2}}{3 + \sqrt{2}} \quad \frac{7}{14 + 6\sqrt{2}}\) 82. \(\frac{1 + \sqrt{7}}{2 - \sqrt{3}}\)

The following exercises are applications of exponentiation and radicals.

83. Business The theory of economic lot size shows that, under certain conditions, the number of units to order to minimize total cost is

\[X = \sqrt{\frac{km}{f}}\]

where \(k\) is the cost to store one unit for one year, \(f\) is the (constant) setup cost to manufacture the product, and \(M\) is the total number of units produced annually. Find \(x\) for the following values of \(f, k, M\).

(a) \(k = \$1, f = \$500, M = 100,000\) 84
(b) \(k = \$3, f = \$7, M = 16,700\) 85
(c) \(k = \$1, f = \$5, M = 18,800\) 86

84. Health The threshold weight \(T\) for a person is the weight above which the risk of death increases greatly. One researcher found that the threshold weight in pounds for men aged 40-49 is related to height in inches by the equation \(h = 12.33t^{0.52}\). What height corresponds to a threshold of 216 pounds for a man in this age group? 73.3 in

Business The annual domestic revenue (in billions of dollars) generated by the sale of movie tickets can be approximated by the function

\[8.19x^{0.406} \quad (x \geq 1)\]

where \(x = 1\) corresponds to 2001. Assuming the model remains accurate, approximate the revenue in the following years. (Data from: www.the-numbers.com.)

85. 2010 About \$10.2 billion 86. 2013 About \$10.5 billion
87. 2015 About \$10.6 billion 88. 2018 About \$10.8 billion

Health The age-adjusted death rates per 100,000 people for diseases of the heart can be approximated by the function

\[262.5x^{-0.56} \quad (x \geq 1)\]

where \(x = 1\) corresponds to 2001. Assuming the model continues to be accurate, find the approximate age-adjusted death rate for the following years. (Data from: U.S. National Center for Health Statistics.)

89. 2011 90. 2013 91. 2017 92. 2020
93. About 188.6 94. About 175.9 95. About 168.7 96. About 164.5

Social Science The number of students receiving financial aid from the federal government in the form of Pell Grants (in millions) can be approximated by the function

\[3.96x^{0.239} \quad (x \geq 1),\]

where \(x = 1\) corresponds to the year 2001. Assuming the model remains accurate, find the number of students receiving a Pell Grant for the following years. (Data from: www.finaid.org.)

93. 2005 94. 2010 95. 2013 96. 2018
97. About 6.7 million 98. About 6.9 million 99. About 7.3 million 100. About 7.9 million

Health A function that approximates the number (in millions) of CT scans performed annually in the United States is

\[3.5x^{1.04} \quad (x \geq 5),\]

where \(x = 5\) corresponds to 1995. Find the approximate number of CT scans performed in the following years. (Data from: The Wall Street Journal.)

98. About 30.4 million 99. About 38.5 million 100. About 47.1 million 101. About 51.3 million

Checkpoint Answers

1. (a) \(2^3\) 2. (b) \((-5)^3\) 3. (c) \((xy)^3\)
4. (a) \(81x^4\) 5. (b) \(r^{12.30}\)
6. (c) \(\frac{32}{x^5}\) 7. (d) \(\frac{9a^4}{b^6}\)
8. (a) \(\frac{1}{36}\) 9. (b) \(-1/216\)
10. (c) \(-1/81\) 11. (d) \(8/5\)
12. (a) \(8/5\) 13. (b) \(2\)
14. (c) \(-16\) 15. (d) \(Not a real number\)
16. (a) \(-1/36\) 17. (b) \(1/216\)
18. (c) \(-1/81\) 19. (d) \(8/5\)
20. (a) \(x^{1/3} y^{2/3}\) 21. (b) \(\frac{6}{x^{1/3} y^{2/3}}\)
22. (c) \(\frac{32}{(0.10k^{7/3})}\) 23. (d) \(\frac{x^2 - x}{xy^2}\)
24. (a) \(\sqrt[3]{\frac{3}{7}}\) 25. (b) \(\sqrt[3]{\frac{3\sqrt{3}}{7}}\)
26. (c) \(11\sqrt{2}\) 27. (d) \(4/5\)
28. (a) \(3\sqrt{5} + \sqrt{10} - 3\sqrt{2} - 2\) 29. (b) \(-4\)
30. (a) \(2\sqrt{5}/5\) 31. (b) \(2 - \sqrt{3}\)
1.6 Exercises

Solve each equation. (See Examples 1–6.)

1. \(3x + 8 = 20\)
2. \(4 - 5y = 19\)
3. \(.6k - .3 = .5k + .4\)
4. \(2.5 + 5.04m = 8.5 - .06m\)
5. \(2a - 1 = 4(a + 1) + 7a + 5\)
6. \(3(k - 2) - 6 = 4k - (3k - 1)\)
7. \(2[x - (3 + 2x) + 9] = 3x - 8\)
8. \(-2[4(k + 2) - 3(k + 1)] = 14 + 2k\)
9. \(\frac{3x}{5} - \frac{4}{5}(x + 1) = 2 - \frac{3}{10}(3x - 4)\)
10. \(\frac{4}{3}(x - 2) - \frac{1}{2} = 2\left(\frac{3}{4}x - 1\right)\)

\[\begin{align*}
11. & \quad \frac{5y}{6} - 8 = 5 - \frac{2y}{3} \\
12. & \quad \frac{x}{2} - 3 = \frac{3x}{5} + 1 \\
13. & \quad \frac{m - 1}{2} = \frac{6m + 5}{12} \\
14. & \quad \frac{3k}{2} + \frac{9k - 5}{6} = \frac{11k + 8}{k} \\
15. & \quad \frac{4}{x - 3} - \frac{8}{2x + 5} + \frac{3}{x - 3} = 0 \\
16. & \quad \frac{5}{2p + 3} - \frac{p - 2}{2p + 3} = \frac{4}{5} \\
17. & \quad \frac{3}{2m + 4} = \frac{1}{m + 2} - 2 \\
18. & \quad \frac{8}{3k - 9} - \frac{5}{k - 3} = 4 \\
\end{align*}\]

Use a calculator to solve each equation. Round your answer to the nearest hundredth. (See Example 3.)

\[\begin{align*}
19. & \quad 9.06x + 3.59(8x - 5) = 12.07x + .5612 \quad x = .72 \\
20. & \quad -5.74(3.1 - 2.7p) = 1.09p + 5.2588 \quad 1.6 \\
21. & \quad \frac{2.63r - 8.99}{1.25} = \frac{3.90r - 1.77}{2.45} = r \quad r = -13.26 \\
22. & \quad \frac{8.19m + 2.55}{4.34} = \frac{8.17m - 9.94}{1.04} = 4m \quad 9.94 \\
\end{align*}\]

Solve each equation for \(x\). (See Example 7.)

\[\begin{align*}
23. & \quad 4(a + x) = b - a + 2x \\
24. & \quad (3a - b) - bx = a(x - 2) \quad (a \neq -b) \\
25. & \quad 5(b - x) = 2b + ax \quad (a \neq -5) \\
26. & \quad bx - 2b = 2a - ax \\
\end{align*}\]

Solve each equation for the specified variable. Assume that all denominators are nonzero. (See Example 7.)

\[\begin{align*}
27. & \quad PV = k \text{ for } V \quad V = \frac{k}{P} \\
28. & \quad i = prt \text{ for } p \quad p = \frac{i}{rt} \\
29. & \quad V = V_0 + gt \text{ for } g \quad g = \frac{V - V_0}{t} \\
30. & \quad S = S_0 + gt^2 + k \text{ for } g \quad g = \frac{S - S_0 - k}{t^2} \\
31. & \quad A = \frac{1}{2}(B + b)h \text{ for } B \quad B = \frac{2A}{h} - b \\
32. & \quad C = \frac{5}{9}(F - 32) \text{ for } F \quad F = \frac{9}{5}C + 32 \\
\end{align*}\]

Solve each equation. (See Examples 8–10.)

\[\begin{align*}
33. & \quad |2h - 1| = 5 \quad h = \frac{3}{2}, \frac{7}{2} \\
34. & \quad |4m - 3| = 12 \quad m = \frac{15}{4}, \frac{21}{4} \\
35. & \quad |6 + 2p| = 10 \quad p = -2, 2 \\
36. & \quad |-5x + 7| = 15 \quad x = \frac{8}{3}, \frac{22}{3} \\
37. & \quad |\frac{5}{r - 3}| = 10 \quad r = \frac{7}{2}, \frac{7}{2} \\
38. & \quad |\frac{3}{2h - 1}| = 4 \quad h = \frac{1}{2}, \frac{1}{8} \\
\end{align*}\]

Solve the following applied problems.

Health \quad According to the American Heart Association, the number \(y\) of brain neurons (in billions) that are lost in a stroke lasting \(x\) hours is given by \(y = \frac{x}{8}\). Find the length of the stroke for the given number of neurons lost.

\[\begin{align*}
39. & \quad 1,250,000,000 \quad x = 10 \\
40. & \quad 2,400,000,000 \quad x = 19,200 \\
\end{align*}\]

Natural Science \quad The equation that relates Fahrenheit temperature \(F\) to Celsius temperature \(C\) is

\[C = \frac{5}{9}(F - 32)\]

Find the Fahrenheit temperature corresponding to these Celsius temperatures.

\[\begin{align*}
41. & \quad -5 \quad 23^\circ \\
42. & \quad -15 \quad 5^\circ \\
43. & \quad 22 \quad 71.6^\circ \\
44. & \quad 36 \quad 96.8^\circ \\
\end{align*}\]

Finance \quad The gross federal debt \(y\) (in trillions of dollars) in year \(x\) is approximated by

\[y = 1.16x + 1.76, \quad \text{where } x \text{ is the number of years after 2000. Assuming the trend continues, in what year will the federal debt be the given amount? (Data from: U.S. Office of Management and Budget.)}\]

\[\begin{align*}
45. & \quad $13.36 trillion \quad 2010 \quad 46. & \quad $16.84 trillion \quad 2013 \\
47. & \quad $19.16 trillion \quad 2015 \quad 48. & \quad $24.96 trillion \quad 2020 \\
\end{align*}\]

Health Economics \quad The total health care expenditures \(E\) in the United States (in trillions of dollars) can be approximated by

\[E = .118x + 1.45, \quad \text{where } x \text{ is the number of years after 2000. Assuming the trend continues, determine the year in which health care expenditures}\]

<sup>6</sup> Indicates answer is in the Additional Instructor Answers at end of the book.
49. $2.63 trillion 2010  50. $2.866 trillion 2012
51. $3.338 trillion 2016  52. $3.574 trillion 2018

Finance  The percentage y (written as a decimal) of U.S. households who owned Roth Individual Retirement Accounts (IRAs) in year x is given by the equation

\[0.09(x - 2004) = 12y - 1.44.\]

Find the year in which the given percentage of U.S. households own a Roth IRA. (Data from: Proquest Statistical Abstract of the United States: 2013.)

53. 18.0% 2012  54. 19.5% 2014
55. 21% 2016  56. 23.25% 2019

Finance  The total amount A (in millions of dollars) donated within a state for charitable contributions claimed on individual federal tax returns can be modeled by the function

\[A = 4.35x - 12,\]

where x is the total number of returns filed (in thousands) for the state. For the given amounts A donated, determine the number of returns that were filed within the following states. (Data from: Proquest Statistical Abstract of the United States: 2013.)

57. California: \(A = 20,777\) million 4779 thousand
58. New York: \(A = 13,732\) million 3160 thousand
59. Texas: \(A = 13,360\) million 3074 thousand
60. Florida: \(A = 9596\) million 2290 thousand

Business  When a loan is paid off early, a portion of the finance charge must be returned to the borrower. By one method of calculating the finance charge (called the rule of 78), the amount of unearned interest (finance charge to be returned) is given by

\[u = f \cdot \frac{n(n + 1)}{q(q + 1)},\]

where \(u\) represents unearned interest, \(f\) is the original finance charge, \(n\) is the number of payments remaining when the loan is paid off, and \(q\) is the original number of payments. Find the amount of the unearned interest in each of the given cases.

61. Original finance charge = $800, loan scheduled to run 36 months, paid off with 18 payments remaining  62. Original finance charge = $1400, loan scheduled to run 48 months, paid off with 12 payments remaining

Business  Solve the following investment problems. (See Example 11.)

63. Joe Gonzalez received $52,000 profit from the sale of some land. He invested part at 5% interest and the rest at 4% interest. He earned a total of $2290 interest per year. How much did he invest at 5%? $21,000
64. Weijen Luan invests $20,000 received from an insurance settlement in two ways: some at 6% and some at 4%. Altogether, she makes $1040 per year in interest. How much is invested at 4%? $8000
65. Maria Martinelli bought two plots of land for a total of $120,000. On the first plot, she made a profit of 15%. On the second, she lost 10%. Her total profit was $5500. How much did she pay for each piece of land? $20,000 for first plot; $53,000 for the second
66. Suppose $20,000 is invested at 5%. How much additional money must be invested at 4% to produce a yield of 4.8% on the entire amount invested? $3000

Solve the given applied problems. (See Example 12.)

According to data from comScore.com, two social media sites, Tumblr.com and Pinterest.com, acquired the same number of unique visitors in 2012. Tumblr.com took 63 months to acquire these visitors while Pinterest.com took 30 months. Pinterest's rate of visitor growth was 450,000 more a month, on average, than Tumblr's average growth per month.

67. Find the average rate of growth of Tumblr.com. About 14,907 per month
68. Find the average rate of growth of Pinterest.com. About 14,907 per month
69. Approximately how many unique visitors did Tumblr.com acquire in 63 months? 25,773,773
70. Approximately how many unique visitors did Pinterest.com acquire in 30 months? 25,773,773

Natural Science  Using the same assumptions about octane ratings as in Example 13, solve the following problems.

71. How many liters of 94-ocetane gasoline should be mixed with 200 liters of 99-octane gasoline to get a mixture that is 97-octane gasoline? 180 liters
72. A service station has 92-octane and 98-octane gasoline. How many liters of each gasoline should be mixed to provide 12 liters of 96-octane gasoline for a chemistry experiment? 4 L of 92-octane gasoline and 8 L of 98-octane gasoline

Solve the following applied problems.

73. Business  A major car rental firm charges $78 a day for a full-size car in Tampa, Florida, with unlimited mileage. Another firm offers a similar car for $55 a day plus 22 cents per mile. How far must you drive in a day in order for the cost to be the same for both vehicles? About 105 miles
74. Business  A car radiator contains 8.5 quarts of fluid, 35% of which is antifreeze. How much fluid should be drained and replaced with pure (100%) antifreeze in order that the new mixture be 65% antifreeze? About 3.9 quarts

Business  Massachusetts has a graduated fine system for speeding, meaning you can pay a base fine and then have more charges added on top. For example, the base fine for speeding is $100. But that is just the start. If you are convicted of going more than 10 mph over the speed limit, add $10 for each additional mph you were traveling over the speed limit plus 10 mph. Thus, the amount of the fine y (in dollars) for driving x miles over the speed limit (when the speed limit is 65 miles per hour) can be represented as

\[y = 10(x - 75) + 100, \quad x \geq 75.\]

(Data from: www.dmv.org.)

75. If Paul was fined $180 for speeding, how fast was he going? 73 mph
76. If Sarah was fined $120 for speeding, how fast was she going? 76 mph
In some applications, it may be necessary to solve an equation in several variables for a specific variable.

**Example 11** Solve \( v = mx^2 + x \) for \( x \). (Assume that \( m \) and \( v \) are positive.)

**Solution** The equation is quadratic in \( x \) because of the \( x^2 \) term. Before we use the quadratic formula, we write the equation in standard form:

\[
v = mx^2 + x = 0 \quad \text{or} \quad mx^2 + x - v = 0.
\]

Let \( a = m \), \( b = 1 \), and \( c = -v \). Then the quadratic formula gives

\[
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.
\]

\[
x = \frac{-1 \pm \sqrt{1^2 - 4m(-v)}}{2m}.
\]

\[
x = \frac{-1 \pm \sqrt{1 + 4mv}}{2m}.
\]

**Checkpoint 10**

Solve each of the given equations for the indicated variable. Assume that all variables are positive.

(a) \( k = \frac{mp^2}{2} - bp \) for \( p \)

(b) \( r = \frac{Ak^2}{3} \) for \( k \)

**1.7 Exercises**

Use factoring to solve each equation. (See Examples 1 and 2.)

1. \((x + 4)(x - 14) = 0\)
2. \((p - 16)(p + 5) = 0\)
3. \(3(x + 6) = 0\)
4. \(x^2 - 2x = 0\)
5. \(2x^2 = 4x\)
6. \(x^2 - 64 = 0\)
7. \(x^2 + 15x + 56 = 0\)
8. \(k^2 - 4k - 5 = 0\)
9. \(2x^2 = 7x - 3\)
10. \(2 = 15z^2 + z\)
11. \(6x^2 + r = 1\)
12. \(3y^2 = 16y - 5\)
13. \(2m^2 + 20 = 13m\)
14. \(6a^2 + 17a + 12 = 0\)
15. \(m(y + 7) = -10\)
16. \(z(2z + 7) = 4\)
17. \(9x^2 - 16 = 0\)
18. \(36y^2 - 49 = 0\)
19. \(16x^2 - 16x = 0\)
20. \(12y^2 - 48y = 0\)

Solve each equation by using the square-root property. (See Example 3.)

21. \((r - 2)^2 = 7\)
22. \((b + 4)^2 = 27\)
23. \((4x - 1)^2 = 20\)
24. \((3t + 5)^2 = 11\)

Use the quadratic formula to solve each equation. If the solutions involve square roots, give both the exact and approximate solutions. (See Examples 4–7.)

25. \(2x^2 + 7x + 1 = 0\)
26. \(3x^2 - x - 7 = 0\)
27. \(4x^2 + 2x = 1\)
28. \(r^2 = 3r + 5\)
29. \(5x^2 + 5y = 2\)
30. \(2z^2 + 3 = 8z\)
31. \(6x^2 + 6x + 4 = 0\)
32. \(3a^2 - 2a + 2 = 0\)
33. \(2r^2 + 3r - 5 = 0\)
34. \(8x^2 = 8x - 3\)
35. \(2x^2 - 7x + 30 = 0\)
36. \(3k^2 + k = 6\)
37. \(1 + \frac{7}{2a} = \frac{15}{2a^2} - \frac{3}{2}\)
38. \(5 - \frac{4}{k} - \frac{1}{k^2} = 0\)

Use the discriminant to determine the number of real solutions of each equation. You need not solve the equations.

39. \(25x^2 + 49 = 70x\)
40. \(9x^2 - 12x = 1\)
41. \(13x^2 + 24x - 5 = 0\)
42. \(20x^2 + 19x + 5 = 0\)

Use a calculator and the quadratic formula to find approximate solutions of each equation. (See Example 5.)

43. \(4.42x^2 - 10.14x + 3.79 = 0\)
44. \(3x^2 - 82.74x + 570.4923 = 0\)
45. \(7.63x^2 + 2.79x = 5.32\)
46. \(8.06x^2 + 25.8726x = 25.047256\)

Solve the following problems. (See Example 3.)

47. **Physical Science** According to the Federal Aviation Administration, the maximum recommended taxiing speed \( s \) (in miles per hour) for a plane on a curved runway exit is given by \( R = 0.5x^2 \), where \( R \) is the radius of the curve (in feet). Find the maximum taxiing speed for planes on such exits when the radius of the exit is

(a) \(450 \text{ ft} \) \hspace{1cm} (b) \(615 \text{ ft} \) \hspace{1cm} (c) \(970 \text{ ft} \)

48. **Social Science** The enrollment \( E \) in public colleges and universities (in millions) is approximated by \( E = 0.011x^2 + 10.7 \) where \( x \) is the number of years since 1990. Find the year when enrollment is the following. (Data from: ProQuest Statistical Abstract of the United States: 2013.)

(a) \(14.7 \text{ million} \) \hspace{1cm} (b) \(17.6 \text{ million} \)

49. **Social Science** The number of traffic fatalities \( F \) (in thousands) where a driver involved in the crash had a blood alcohol level of .08 or higher can be approximated by the function

\[ F = -0.079x^2 + 0.46x + 13.3 \quad (0 \leq x \leq 10), \]

where \( x = 0 \) corresponds to the year 2000. Determine in what year the number of fatalities is approximately the given number. (Data from: National Highway Traffic Safety Administration.)

(a) \(12,600 \) \hspace{1cm} (b) \(11,000 \)

\( * \) indicates answer is in the Additional Instructor Answers at the end of the book.
50. **Finance** The total assets $A$ of private and public pension funds (in trillions of dollars) can be approximated by the function

$$A = 0.237x^2 - 3.96x + 28.2 \quad (6 \leq x \leq 11),$$

where $x = 6$ corresponds to the year 2006. (Data from: Board of Governors of the Federal Reserve System.)

(a) What were the assets in 2008? About $14.7$ trillion

(b) What year after 2008 produced $12.3$ trillion in assets? 2010

51. **Finance** The assets $A$ of public pension funds (in trillions of dollars) can be approximated by the function

$$A = 0.169x^2 - 2.85x + 19.6 \quad (6 \leq x \leq 11),$$

where $x = 6$ corresponds to the year 2006. (Data from: Board of Governors of the Federal Reserve System.)

(a) What were the assets in 2009? About $37.6$ trillion

(b) What year before 2008 produced $7.9$ trillion in assets? 2007

52. **Business** The net income for Apple $A$ (in billions) can be approximated by the function

$$A = 8.77x^2 - 9.33x + 23.4 \quad (6 \leq x \leq 12),$$

where $x = 6$ corresponds to 2006. Find the year in which net income was the following. (Data from: www.morningstar.com.)

(a) $17.8$ billion 2010

(b) $37.7$ billion 2012

Solve the following problems. (See Examples 8 and 9.)

53. **Physical Science** A 13-foot-long ladder leans on a wall, as shown in the accompanying figure. The bottom of the ladder is 5 feet from the wall. If the bottom is pulled out 2 feet farther from the wall, how far does the top of the ladder move down the wall? *[Hint: Draw pictures of the right triangle formed by the ladder, the ground, and the wall before and after the ladder is moved. In each case, use the Pythagorean theorem to find the distance from the top of the ladder to the ground.]* About 1.046 ft

54. **Physical Science** A 15-foot-long pole leans against a wall. The bottom is 9 feet from the wall. How much farther should the bottom be pulled away from the wall so that the top moves the same amount down the wall? 3 ft

55. **Physical Science** Two trains leave the same city at the same time, one going north and the other east. The eastbound train travels 20 mph faster than the northbound one. After 5 hours, the trains are 300 miles apart. Determine the speed of each train, using the following steps.

(a) Let $x$ denote the speed of the northbound train. Express the speed of the eastbound train in terms of $x$. $x + 20$

(b) Write expressions that give the distance traveled by each train after 5 hours. Northbound: $5x$; eastbound: $5(x + 20)$ or $5x + 100$

(c) Use part (b) and the fact that the trains are 300 miles apart after 5 hours to write an equation. (A diagram of the situation may help.) $(5x)^2 + (5x + 100)^2 = 300^2$

(d) Solve the equation and determine the speeds of the trains. About 31.23 mph and 51.23 mph

56. **Physical Science** Chris and Josh have received walkie-talkies for Christmas. If they leave from the same point at the same time, Chris walking north at 2.5 mph and Josh walking east at 3 mph, how long will they be able to talk to each other if the range of the walkie-talkies is 4 miles? Round your answer to the nearest minute. About 61 min

57. **Physical Science** An ecology center wants to set up an experimental garden. It has 300 meters of fencing to enclose a rectangular area of 5000 square meters. Find the length and width of the rectangle as follows.

(a) Let $x =$ the length and write an expression for the width. $5000 - 2x$

(b) Write an equation relating the length, width, and area, using the result of part (a). $x(5000 - 2x) = 5000$

(c) Solve the problem. Length 100 m; width 50 m

58. **Business** A landscape architect has included a rectangular flower bed measuring 9 feet by 5 feet in her plans for a new building. She wants to use two colors of flowers in the bed, one in the center and the other for a border of the same width on all four sides. If she can get just enough plants to cover 24 square feet for the border, how wide can the border be? 6 in

59. **Physical Science** Joan wants to buy a rug for a room that is 12 feet by 15 feet. She wants to leave a uniform strip of floor around the rug. She can afford 108 square feet of carpeting. What dimensions should the rug have? 9 by 12 ft

60. **Physical Science** In 2012, Dario Franchitti won the 500-mile Indianapolis 500 race. His speed (rate) was, on average, 92 miles per hour faster than that of the 1911 winner, Ray Harroun. Franchitti completed the race in 3.72 hours less time than Harroun. Find Harroun’s and Franchitti’s rates to the nearest tenth. Harroun: 74.3 mph; Franchitti: 166.3 mph

**Physical Science** Use the height formula in Example 10 to work the given problems. Note that an object that is dropped (rather than thrown downward) has initial velocity $v_0 = 0$.

61. How long does it take a baseball to reach the ground if it is dropped from the top of a 625-foot-high building? Compare the answer with that in Example 10. 6.25 sec

62. After the baseball in Exercise 61 is dropped, how long does it take for the ball to fall 196 feet? (Hint: How high is the ball at that time?) 3.5 sec

63. You are standing on a cliff that is 200 feet high. How long will it take a rock to reach the ground if

(a) you drop it? About 3.54 sec

(b) you throw it downward at an initial velocity of 40 feet per second? 2.5 sec

(c) How far does the rock fall in 2 seconds if you throw it downward with an initial velocity of 40 feet per second? 144 ft

64. A rocket is fired straight up from ground level with an initial velocity of 800 feet per second.

(a) How long does it take the rocket to rise 3200 feet? About 4.38 sec

(b) When will the rocket hit the ground? 50 sec