The most common examples of simple discount loans are U.S. Treasury bills (T-bills), which are essentially short-term loans to the U.S. government by investors. T-bills are sold at a discount from their face value and the Treasury pays back the face value of the T-bill at maturity. The discount amount is the interest deducted in advance from the face value. The Treasury receives the face value less the discount, but pays back the full face value.

Example 8
Finance
An investor bought a six-month $8000 treasury bill on February 28, 2013 that sold at a discount rate of .135%. What is the amount of the discount? What is the price of the T-bill? (Data from: www.treasurydirect.gov.)

Solution
The discount rate on a T-bill is always a simple annual interest rate. Consequently, the discount (interest) is found with the simple interest formula, using $P = 8000$ (face value), $r = .00135$ (discount rate), and $t = .5$ (because 6 months is half a year):

\[
\text{Discount} = Prt = 8000 \times .00135 \times .5 = 5.40.
\]

So the price of the T-bill is

\[
\text{Face Value} - \text{Discount} = 8000 - 5.40 = 7994.60.
\]

In a simple discount loan, such as a T-bill, the discount rate is not the actual interest rate the borrower pays. In Example 8, the discount rate .135% was applied to the face value of $8000, rather than the $7994.60 that the Treasury (the borrower) received.

Example 9
Finance
Find the actual interest rate paid by the Treasury in Example 8.

Solution
Use the formula for simple interest, $I = Prt$ with $r$ as the unknown. Here, $P = 7994.60$ (the amount the Treasury received) and $I = 5.40$ (the discount amount). Since this is a six-month T-bill, $t = .5$, and we have

\[
I = Prt
\]

\[
5.40 = 7994.60(r)(.5)
\]

\[
5.40 = 3997.3r
\]

\[
r = \frac{5.40}{3997.3} = .0013509.
\]

Divide both sides by 3997.3.

Multiply out right side.

So the actual interest rate is .13509%.

Checkpoint 9

Find the actual interest rate paid by the Treasury for each T-bill in Example 8.

5.1 Exercises

Unless stated otherwise, "interest" means simple interest, and "interest rate" and "discount rate" refer to annual rates. Assume 365 days in a year.

1. What factors determine the amount of interest earned on a fixed principal? Time and interest rate.

Find the interest on each of these loans. (See Example 1.)

2. $35,000 at 6% for 9 months $1575
3. $2850 at 7% for 8 months $133
4. $1875 at 5.3% for 7 months $57.99
5. $3650 at 6.5% for 11 months $217.48
6. $5160 at 7.1% for 58 days $20.22
7. $2830 at 8.9% for 125 days $86.26
8. $8940 at 9%; loan made on May 7 and due September 19 $392.39
9. $5328 at 8%; loan made on August 16 and due December 30 $166.66
10. $7900 at 7%; loan made on July 17 and due October 25 $307.68

Finance For each of the given corporate bonds, whose interest rates are provided, find the semiannual interest payment and the total interest earned over the life of the bond. (See Example 2. Data from: www.finra.org.)

11. $5000 IBM, 3-year bond; 1.25% $31.25; $107.50
12. $9000 Barrick Gold Corp., 10-year bond; 3.85% $173.25; $3465.06
13. $12,500 Morgan Stanley, 10-year bond; 3.75% $234.38; $4687.51
14. $4500 Goldman Sachs, 3-year bond; 6.75% $4513.35; $911.25
15. $6500 Amazon.com Corp, 10-year bond; 2.5% $61125; $1605
16. $10,000 Wells Fargo, 10-year bond; 3.45% $112350; $3450

Find the future value of each of these loans. (See Example 3.)

17. $12,000 loan at 3.5% for 3 months $12130
18. $3475 loan at 7.5% for 6 months $3681.34
19. $6500 loan at 5.25% for 8 months $6772.99
20. $24,500 loan at 9.6% for 10 months $26460

21. What is meant by the present value of money?

22. In your own words, describe the maturity value of a loan.

Find the present value of each future amount. (See Examples 5 and 6.)

23. $15,000 for 9 months; money earns 6% $14354.07
24. $48,000 for 8 months; money earns 5% $46451.61
25. $15,402 for 120 days; money earns 6.3% $15089.46
26. $29,764 for 310 days; money earns 7.2% $29042.89

Finance The given treasury bills were sold on April 4, 2013. Find (a) the price of the T-bill, and (b) the actual interest rate paid by the Treasury. (See Examples 8 and 9. Data from: www.treasurydirect.gov.)

27. Three-month $20,000 T-bill with discount rate of .075% $19927.86; about 0.2%
28. One-month $12,750 T-bill with discount rate of .070% $12499.26; about 0.3%
29. Six-month $15,500 T-bill with discount rate of .05% $15251.32; about 0.0%
30. One-year $7000 T-bill with discount rate of .140% $6999.25; about 0.1%

Finance Historically, treasury bills offered higher rates. On March 9, 2007 the discount rates were substantially higher than in April, 2013. For the following treasury bills bought in 2007, find (a) the price of the T-bill, and (b) the actual interest rate paid by the Treasury. (See Examples 8 and 9. Data from: www.treasury.gov.)

31. Three-month $20,000 T-bill with discount rate of 4.96% $19737.42; about 5.0%
32. One-month $12,750 T-bill with discount rate of 5.13% $12077.77; about 5.2%
33. Six-month $15,500 T-bill with discount rate of 4.93% $15117.96; about 5.0%
34. Six-month $9000 T-bill with discount rate of 4.93% $8778.15; about 5.0%

Finance Work the following applied problems.

35. In March 1868, Winston Churchill’s grandfather, L.W. Jerome, issued $1000 bonds (to pay for a road to a race track he owned in what is now the Bronx). The bonds carried a 7% annual interest rate payable semiannually. Mr. Jerome paid the interest until March 1874, at which time New York City assumed responsibility for the bonds (and the road they financed). (Data from: New York Times, February 13, 2009.)

(a) The first of these bonds matured in March 2009. At that time, how much interest had New York City paid on this bond? $9459
(b) Another of these bonds will not mature until March 2147! At that time, how much interest will New York City have paid on it? $39,110

36. An accountant for a corporation forgot to pay the firm’s income tax of $725,896.15 on time. The government charged a penalty of 9.8% interest for the 34 days the money was late. Find the total amount (tax and penalty) that was paid. $732,522.69

37. Mike Branson invested his summer earnings of $3000 in a savings account for college. The account pays 2.5% interest. How much will this amount to in 9 months? $3095.25

38. To pay for textbooks, a student borrows $450 from a credit union at 6.5% simple interest. He will repay the loan in 38 days, when he expects to be paid for tutoring. How much interest will he pay? $10.95

39. An account invested in a money market fund grew from $67,081.20 to $67,359.39 in a month. What was the interest rate, to the nearest tenth? 0.4%

40. A $100,000 certificate of deposit held for 60 days is worth $101,133.33. To the nearest tenth of a percent, what interest rate was earned? \( y = 6.9\% \)

41. Dave took out a $7500 loan at 7% and eventually repaid $7675 (principal and interest). What was the time period of the loan? 4 months

42. What is the time period of a $10,000 loan at 6.75%, in which the total amount of interest paid was $618.75? 11 months

43. Tuition of $1769 will be due when the spring term begins in 4 months. What amount should a student deposit today, at 3.25%, to have enough to pay the tuition? $1750.04

44. A firm of accountants has ordered 7 new computers at a cost of $5104 each. The machines will not be delivered for 7 months. What amount could the firm deposit in an account paying 6.42% to have enough to pay for the machines? $31,443.29

45. John Sun Yee needs $6000 to pay for remodeling work on his house. A contractor agrees to do the work in 10 months. How much should Yee deposit at 3.6% to accumulate the $6000 at that time? $5825.24

46. Lorie Reilly decides to go back to college. For transportation, she borrows money from her parents to buy a small car for $7200. She plans to repay the loan in 7 months. What amount can she deposit today at 5.25% to have enough to pay off the loan? $6986.85

47. A six-month $4000 Treasury bill sold for $3930. What was the discount rate? 2.5%

48. A three-month $7600 Treasury bill carries a discount of $80.75. What is the discount rate for this T-bill? 4.29%

Finance Work the next set of problems, in which you are to find the annual simple interest rate. Consider any fees, dividends, or profits as part of the total interest.

49. A stock that sold for $22 at the beginning of the year was selling for $24 at the end of the year. If the stock paid a dividend of $0.50 per share, what is the simple interest rate on an investment in this stock? (Hint: Consider the interest to be the increase in value plus the dividend.) About 11.39%
5.2 Exercises

Interest on the zero-coupon bonds here is compounded semiannually.

1. In the preceding summary what is the difference between \( r \) and \( r' \)? Between \( r \) and \( r' \)?

2. Explain the difference between simple interest and compound interest. Answer: very.

3. What factors determine the amount of interest earned on a fixed principal? Interest rate and number of compounding periods.

4. In your own words, describe the maturity value of a loan. Answer: very.

5. What is meant by the present value of money? Effective rate.

6. If interest is compounded more than once per year, which rate is higher, the stated rate or the effective rate? Effective rate.

Find the compound amount and the interest earned for each of the following deposits. (See Examples 1, 2, 4, and 5.)

7. $1000 at 4% compounded annually for 6 years $1265.32; $265.32.

8. $1000 at 6% compounded annually for 10 years $1995.25; $995.25.

9. $470 at 8% compounded semiannually for 12 years $759.37; $259.37.

10. $15,000 at 4.6% compounded semiannually for 11 years $26,727.47; $16,727.47.

11. $450 at 4.5% compounded quarterly for 8 years $5297.03; $5297.03.

12. $910 at 6.1% compounded quarterly for 4 years $1,249.36; $349.36.

Finance The following CD rates were available on www.bankrate.com on April 13, 2013. Find the compound amount and the interest earned for each of the following. (See Example 5.)

13. Virtual Bank: $10,000 at 0.9% compounded daily for 1 year $10,090.41; $90.41.

14. AloStar Bank of Commerce: $1000 at .85% compounded daily for 1 year $1009.54; $9.54.

15. USA; $5000 at .81% compounded monthly for 2 years $5108.63; $108.63.

16. Centennial Bank: $20,000 at .45% compounded monthly for 2 years $20,180.78; $180.78.

17. E-LOAN: $100,000 at 1.52% compounded daily for 5 years $110,233.00; $2,333.00.

18. Third Federal Savings and Loans: $150,000 at 1.15% compounded quarterly for 5 years $158,864.69; $864.69.

Find the interest rate (with annual compounding) that makes the statement true. (See Example 3.)

19. $3000 grows to $3606 in 5 years 3.75%.

20. $2550 grows to $3095 in 11 years 3.91%.

21. $8500 grows to $12,161 in 7 years 5.29%.

22. $9000 grows to $17,118 in 16 years 4.10%.

Find the compound amount and the interest earned when the following investments have continuous compounding. (See Example 6.)

23. $20,000 at 3.5% for 5 years $23,324.92; $3,324.92.

24. $15,000 at 2.9% for 10 years $20,046.41; $5,046.41.

25. $30,000 at 1.8% for 3 years $31,664.54; $1,664.54.

26. $100,000 at 5.1% for 20 years $277,319.48; $177,319.48.

Find the face value (to the nearest dollar) of the zero-coupon bond. (See Example 7.)

27. 15-year bond at 5.2%; price $4630 $10,000.

28. 10-year bond at 4.1%; price $13,328 $20,000.

29. 20-year bond at 3.5%; price $9992 $20,000.

30. How do the nominal, or stated, interest rate and the effective interest rate (APY) differ? Answers vary.

Find the APY corresponding to the given nominal rates. (See Examples 9–11.)

31. 4% compounded semiannually 4.04%.

32. 6% compounded quarterly 6.136%.

33. 5% compounded quarterly 5.095%.

34. 4.7% compounded semiannually 4.775%.

Find the present value of the given future amounts. (See Example 12.)

35. $12,000 at 5% compounded annually for 6 years $8954.58.

36. $8500 at 6% compounded annually for 9 years $5901.14.

37. $17,230 at 4% compounded quarterly for 10 years $11,572.58.

38. $5240 at 6% compounded quarterly for 8 years $3354.

What price should you be willing to pay for each of these zero-coupon bonds? (See Example 13.)

39. 5-year $5000 bond; interest at 3.5% $4203.64.

40. 10-year $10,000 bond; interest at 4% $6729.21.

41. 15-year $20,000 bond; interest at 4.7% $9933.10.

42. 20-year $15,000 bond; interest at 5.3% $5269.06.

Finance For Exercises 43 and 44, assume an annual inflation rate of 2.07% (the annual inflation rate of 2012 according to www.InflationData.com). Find the previous price of the following items. (See Example 14.)

43. How much did an item that costs $5000 now cost 4 years prior? $4596.21.

44. How much did an item that costs $7500 now cost 5 years prior? $6709.72.

45. If the annual inflation rate is 3.6%, how much did an item that costs $500 now cost 2 years prior? $465.35.

46. If the annual inflation rate is 1.18%, how much did an item that costs $1250 now cost 6 years prior? $1165.08.

47. If money can be invested at 8% compounded quarterly, which is larger, $1000 now or $1210 in 5 years? Use present value to decide. $1000 now.

48. If money can be invested at 6% compounded annually, which is larger, $10,000 now or $15,000 in 6 years? Use present value to decide. $15,000 in 6 years.
Finance  Work the following applied problems.

49. A small business borrows $30,000 for expansion at 9% compounded monthly. The loan is due in 4 years. How much interest will the business pay? $2,137.27

50. A developer needs $80,000 to buy land. He is able to borrow the money at 10% per year compounded quarterly. How much will he pay if he pays off the loan in 5 years? $51,000.32

51. Lora Reilly has inherited $10,000 from her uncle’s estate. She will invest the money for 2 years. She is considering two investments: a money market fund that pays a guaranteed 5.8% interest compounded daily and a 2-year Treasury note at 6% annual interest. Which investment pays the most interest over the 2-year period? 2-year Treasury note

52. Which of these 20-year zero-coupon bonds will be worth more at maturity: one that sells for $4510, with a 6.1% interest rate, or one that sells for $5809, with a 4.8% interest rate? The 4.8% one (by $1109)

53. As the prize in a contest, you are offered $1000 now or $1210 in 5 years. If money can be invested at 6% compounded annually, which is larger? $1210 now

54. Two partners agree to invest equal amounts in their business. One will contribute $10,000 immediately. The other plans to contribute an equivalent amount in 3 years, when she expects to acquire a large sum of money. How much should she contribute at that time to match her partner’s investment now, assuming an interest rate of 6% compounded semiannually? $11,940.52

55. In the Capital Appreciation Fund, a mutual fund from T. Rowe Price, a $10,000 investment grew to $11,115 over the 3-year period 2010–2013. Find the annual interest rate, compounded yearly, that this investment earned. 3.96%

56. In the Vanguard Information Technology Index Fund, a $10,000 investment grew to $16,904.75 over the 10-year period 2003–2013. Find the annual interest rate, compounded yearly, that this investment earned. 5.96%

57. The Flagstar Bank in Michigan offered a 5-year certificate of deposit (CD) at 4.38% interest compounded quarterly in June 2005. On the same day, the Internet, Principal Bank offered a 5-year CD at 4.37% interest compounded monthly. Find the APY for each CD. Which bank paid a higher APY? The Internet, Principal paid a higher APY

58. The Westfield Bank in Ohio offered the CD rates shown in the accompanying table in October 2008. The APY rates shown assume monthly compounding. Find the corresponding nominal rates to the nearest hundredth. (Hint: Solve the effective-rate equation for r.)

<table>
<thead>
<tr>
<th>Term</th>
<th>6 mo</th>
<th>1 yr</th>
<th>2 yr</th>
<th>3 yr</th>
<th>5 yr</th>
</tr>
</thead>
<tbody>
<tr>
<td>APY (%)</td>
<td>2.25%</td>
<td>2.50%</td>
<td>3.00%</td>
<td>3.25%</td>
<td>3.75%</td>
</tr>
</tbody>
</table>

59. A company has agreed to pay $2.9 million in 5 years to settle a lawsuit. How much must it invest now in an account paying 5% interest compounded monthly to have that amount when it is due? About $2,290,400

60. Bill Poole wants to have $20,000 available in 5 years for a down payment on a house. He has inherited $16,000. How much of the inheritance should he invest now to accumulate the $20,000 if he can get an interest rate of 5.5% compounded quarterly? $17,572.74

61. If inflation has been running at 3.75% per year and a new car costs $23,500 today, what would it cost three years ago? About $21,504.70

62. If inflation is 2.4% per year and a washing machine costs $345 today, what did a similar model cost five years ago? About $300

Economics  Use the approach in Example 8 to find the time it would take for the general level of prices in the economy to double at the average annual inflation rates in Exercises 63–66.

63. 3% 65. 5% 67. The consumption of electricity has increased historically at 6% per year. If the annual rate grows at this rate indefinitely, how many years will it take for the consumption of electricity to double? About 23 years

64. 4% 66. 5.5% 68. Suppose a conservation campaign coupled with higher rates caused the demand for electricity to increase at only 2% per year, as it has recently. The number of years before the utility companies will need to double their generating capacity. 35 years

69. You decide to invest a $16,000 bonus in a money market fund that guarantees a 5.5% annual interest rate compounded monthly for 7 years. A one-time fee of $30 is charged to set up the account. In addition, there is an annual administrative charge of 1.25% of the balance in the account at the end of each year.
   (a) How much is in the account at the end of the first year? $16,659.95
   (b) How much is in the account at the end of the seventh year? $25,472.27

70. Joe Marus decide to invest $12,000 in a money market fund that guarantees a 4.6% annual interest rate compounded daily for 6 years. A one-time fee of $25 is charged to set up the account. In addition, there is an annual administration charge of .9% of the balance in the account at the end of each year.
   (a) How much is in the account at the end of the first year? $12,497.72
   (b) How much is in the account at the end of the sixth year? $15,126.06

The following exercises are from professional examinations.

71. On January 1, 2002, Jack deposited $1000 into Bank X to earn interest at the rate of j per annum compounded semiannually. On January 1, 2007, he transferred his account to Bank Y to earn interest at the rate of k per annum compounded quarterly. On January 1, 2010, the balance at Bank Y was $1990.76. If Jack could have earned interest at the rate of k per annum compounded quarterly from January 1, 2002, through January 1, 2010, his balance would have been $2203.76. Which of the following represents the ratio k/j? (Deposit of Jack in Bank X from Course 140 Examination, Mathematics of Compound Interest. Copyright © Society of Actuaries. Reproduced by permission of Society of Actuaries.)
   (a) 1.25  (b) 1.30  (c) 1.35  (d) 1.40  (e) 1.45

72. On January 1, 2009, Tone Company exchanged equipment for a $200,000 non-interest-bearing note due on January 1, 2012. The prevailing rate of interest for a note of this type on January 1, 2009, was 10%. The present value of $1 at 10% for three periods is 0.75. What amount of interest revenue should be included in Tone’s 2010 income statement? (Adapted from the Uniform CPA Examination, American Institute of Certified Public Accountants.)
   (a) $7500  (b) $15,000  (c) $16,500  (d) $20,000
5.3 Annuities, Future Value, and Sinking Funds

Checkpoint 6

(a) Ms. Black deposits $800 at the beginning of each 6-month period for 5 years. Find the final amount if the account pays 6% compounded semiannually.

(b) Find the final amount if this account were an ordinary annuity.

Solution

In 7 years, there are \( n = 28 \) quarterly periods. For an annuity due, add one period to get \( n + 1 = 29 \), and use the formula with \( i = .08/4 = .02 \):

\[
S = R \left( \frac{(1 + i)^{n+1} - 1}{i} \right) = 500 \left( \frac{(1 + .02)^{29} - 1}{.02} \right) = 500 = 18,896.12.
\]

After 7 years, the account balance will be $18,896.12. 

Example 8

Jay Rechtien plans to have a fixed amount from his paycheck directly deposited into an account that pays 5.5% interest, compounded monthly. If he gets paid on the first day of the month and wants to accumulate $13,000 in the next three-and-a-half years, how much should he deposit each month?

Solution

Jay's deposits form an annuity due whose future value is \( S = 13,000 \). The interest rate is \( i = .055/12 \). There are 42 months in three-and-a-half years. Since this is an annuity due, add one period, so that \( n + 1 = 43 \). Then solve the future-value formula for the payment \( R \):

\[
R \left( \frac{(1 + i)^{n+1} - 1}{i} \right) = S
\]

\[
R \left( \frac{(1 + .055/12)^{43} - 1}{.055/12} \right) = 13,000
\]

Factor out \( R \) on left side.

\[
R \left( 46.4103 \right) = 13,000
\]

Computes left side.

\[
R = \frac{13,000}{46.4103} = 280.110
\]

Divide both sides by 46.4103.

Jay should have $280.11 deposited from each paycheck.

\[\text{Figure 5.12}\]

5.3 Exercises

Note: Unless stated otherwise, all payments are made at the end of the period.

Find each of these sums (to 4 decimal places).

1. \( 1 + 1.05 + 1.05^2 + 1.05^3 + \cdots + 1.05^{14} \) 21,5784

2. \( 1 + 1.046 + 1.046^2 + 1.046^3 + \cdots + 1.046^{21} \) 36,7317

Find the future value of the ordinary annuities with the given payments and interest rates. (See Examples 1, 2, 3(a), and 4.)

3. \( R = 12,000, 6.2\% \) interest compounded annually for 8 years 519,625.61

4. \( R = 20,000, 4.5\% \) interest compounded annually for 12 years 287,358.89

5. \( R = 865, 6\% \) interest compounded semiannually for 10 years 223,392.37

6. \( R = 7300, 9\% \) interest compounded semiannually for 6 years 117,887.41

7. \( R = 1200, 8\% \) interest compounded quarterly for 10 years 127,648.35

8. \( R = 20,000, 6\% \) interest compounded quarterly for 12 years 51,195,304.39

Find the final amount (rounded to the nearest dollar) in each of these retirement accounts, in which the rate of return on the account and the regular contribution change over time. (See Example 3.)

9. $400 per month invested at 4%, compounded monthly, for 10 years; then $600 per month invested at 6%, compounded monthly, for 10 years. About $205,490

10. $500 per month invested at 5%, compounded monthly, for 20 years; then $1000 per month invested at 8%, compounded monthly, for 20 years. About $310,831

11. $1000 per quarter invested at 4.2%, compounded quarterly, for 10 years; then $1500 per quarter invested at 7.4%, compounded quarterly, for 15 years. About $310,831

12. $1500 per quarter invested at 7.4%, compounded quarterly, for 15 years; then $1000 per quarter invested at 4.2%, compounded quarterly, for 10 years. (Compare with Exercise 11.) About $256,115

Find the amount of each payment to be made into a sinking fund to accumulate the given amounts. Payments are made at the end of each period. (See Example 5.)

13. $11,000; money earns 5% compounded semiannually for 6 years $2797.36
14. $65,000; money earns 6% compounded semiannually for 4\(\frac{1}{2}\) years
   $689,359.60
15. $50,000; money earns 8% compounded quarterly for 2\(\frac{1}{2}\) years
   $456,333.44
16. $25,000; money earns 9% compounded quarterly for 3\(\frac{1}{2}\) years
   $159,069.26
17. $6000; money earns 6% compounded monthly for 3 years
   $192,534.04
18. $9000; money earns 7% compounded monthly for 2\(\frac{1}{2}\) years
   $275,795.79

Find the interest rate needed for the sinking fund to reach the required amount. Assume that the compounding period is the same as the payment period. (See Example 6.)

19. $50,000 to be accumulated in 10 years; annual payments of $3940. 5.19%
20. $100,000 to be accumulated in 15 years; quarterly payments of $1200. 4.25%
21. $38,000 to be accumulated in 5 years; quarterly payments of $1675. 5.22%
22. $77,000 to be accumulated in 20 years; monthly payments of $195. 4.69%

23. What is meant by a sinking fund? List some reasons for establishing a sinking fund. Answer may.

24. Explain the difference between an ordinary annuity and an annuity due. Answer may.

Find the future value of each annuity due. (See Example 7.)

25. Payments of $500 for 10 years at 5% compounded annually
   $6283.39
26. Payments of $1050 for 8 years at 3.5% compounded annually
   $9902.34
27. Payments of $16,000 for 11 years at 4.7% compounded annually
   $32,928.32
28. Payments of $25,000 for 12 years at 6% compounded annually
   $47,653.64
29. Payments of $1000 for 9 years at 8% compounded semiannually
   $12,289.16
30. Payments of $750 for 15 years at 6% compounded semiannually
   $13,722.61
31. Payments of $100 for 7 years at 9% compounded quarterly
   $735.85
32. Payments of $1500 for 11 years at 7% compounded quarterly
   $19,875.82

Find the payment that should be used for the annuity due whose future value is given. Assume that the compounding period is the same as the payment period. (See Example 8.)

33. $8000; quarterly payments for 3 years; interest rate 4.4%
   $2602.86
34. $12,000; annual payments for 6 years; interest rate 5.1%
   $11243.89
35. $35,000; monthly payments for 12 years; interest rate 5.7%
   $3603.21
36. $125,000; monthly payments for 9 years; interest rate 6%
   $1271.36

Finance Work the following applied problems.

37. A typical pack-a-day smoker in Ohio spends about $170 per month on cigarettes. Suppose the smoker invests that amount at the end of each month in an investment fund that pays a return of 5.3% compounded monthly. What would the account be worth after 40 years? (Data from: www.theawl.com.)
   $700,106.52
38. A typical pack-a-day smoker in Illinois spends about $307.50 per month on cigarettes. Suppose the smoker invests that amount at the end of each month in an investment fund that pays a return of 4.9% compounded monthly. What would the account be worth after 40 years? (Data from: www.theawl.com.)
   $457,187.25
39. The Vanguard Explorer Value fund had as of April 2013 a 10-year average return of 10.99%. (Data from: www.vanguard.com.)
   (a) If Becky Anderson deposited $800 a month in the fund for 10 years, find the final value of the amount of her investments. Assume monthly compounding.
   $173,497.36
   (b) If Becky had invested instead with the Vanguard Growth and Income fund, which had an average annual return of 7.77%, what would the final value of the amount of her investments be? Assume monthly compounding.
   $144,493.82
   (c) How much more did the Explorer Value fund generate than the Growth and Income fund? $29,004.04
40. The Janus Enterprise fund had as of April 2013 a 10-year average return of 12.54%. (Data from: www.janus.com.)
   (a) If Elaine Chuha deposited $625 a month in the fund for 8 years, find the final value of the amount of her investments. Assume monthly compounding.
   $400,442.33
   (b) If Elaine had invested instead with the Janus Twenty fund, which had an average annual return of 10.63%, what would the final value of the amount of her investments be? Assume monthly compounding.
   $319,967.67
   (c) How much more did the Janus Enterprise fund generate than the Janus Twenty fund? $84,474.66
41. Brian Feister, a 25-year-old professional, invests $200 a month in the T. Rowe Price Capital Opportunity fund, which has a 10-year average return of 8.75%. (Data from: www.troweprice.com.)
   (a) Brian wants to estimate what he will have for retirement when he is 60 years old if the rate stays constant. Assume monthly compounding.
   $552,599.56
   (b) If Brian makes no further deposits and makes no withdrawals after age 60, how much will he have for retirement at age 65? $834,433.78
42. Ian Morrison, a 30-year-old professional, invests $250 a month in the T. Rowe Price Equity Income fund, which has a 10-year average return of 9.04%. (Data from: www.troweprice.com.)
   (a) Ian wants to estimate what he will have for retirement when he is 65 years old if the rate stays constant. Assume monthly compounding.
   $747,081.31
   (b) If Ian makes no further deposits and makes no withdrawals after age 65, how much will he have for retirement at age 75? Assume monthly compounding.
   $1,428,361.37
43. A mother opened an investment account for her son on the day he was born, investing $1000. Each year on his birthday, she deposits another $1000, making the last deposit on his 18th birthday. If the account paid a return rate of 5.6% compounded annually, how much is in the account at the end of the day on the son’s 18th birthday? $23,416.46
5.3 Annuities, Future Value, and Sinking Funds

44. A grandmother opens an investment account for her only grandchild on the day she was born, investing $500. Each year on her birthday, she deposits another $500, making the last deposit on her 25th birthday. If the account paid a return rate of 6.2% compounded annually, how much is in the account at the end of the day on the grandchild's 25th birthday? $30,466.92

45. Chuck Hickman deposits $10,000 at the beginning of each year for 12 years in an account paying 5% compounded annually. He then puts the total amount on deposit in another account paying 6% compounded semi-annually for another 9 years. Find the final amount on deposit after the entire 21-year period. $394,377.35

46. Suppose that the best rate that the company in Example 6 can find is 6.3%, compounded monthly (rather than the 6.661% it wants). Then the company must deposit more in the sinking fund each month. What monthly deposit will guarantee that the fund will be worth $100,000 in 20 years? $394,377.35

47. David Horwitz needs $10,000 in 8 years.
(a) What amount should he deposit at the end of each quarter at 5% compounded quarterly so that he will have his $10,000? $226.38
(b) Find Horwitz's quarterly deposit if the money is deposited at 5.8% compounded quarterly. $247.31

48. Harv's Meats knows that it must buy a new machine in 4 years. The machine costs $12,000. In order to accumulate enough money to pay for the machine, Harv decides to deposit a sum of money at the end of each 6 months in an account paying 6% compounded semiannually. How much should each payment be? $1349.48

49. Barbara Margolis wants to buy a $24,000 car in 6 years. How much money must she deposit at the end of each quarter in an account paying 5% compounded quarterly so that she will have enough to pay for her car? $1063.64

50. The Chinn's agree to sell an antique vase to a local museum for $19,000. They want to defer the receipt of this money until they retire in 5 years (and are in a lower tax bracket). If the museum can earn 5.8%, compounded annually, find the amount of each annual payment it should make into a sinking fund so that it will have the necessary $19,000 in 5 years. $3384.02

51. Diane Gray sells some land in Nevada. She will be paid a lump sum of $60,000 in 7 years. Until then, the buyer pays 8% simple interest quarterly.
(a) Find the amount of each quarterly interest payment. $1200
(b) The buyer sets up a sinking fund so that enough money will be present to pay off the $60,000. The buyer wants to make semiannual payments into the sinking fund; the account pays 6% compounded semiannually. Find the amount of each payment into the fund. $3311.58

52. Joe Seniw bought a rare stamp for his collection. He agreed to pay a lump sum of $4000 after 5 years. Until then, he pays 6% simple interest semiannually.
(a) Find the amount of each semiannual interest payment. $120
(b) Seniw sets up a sinking fund so that enough money will be present to pay off the $4000. He wants to make semiannual payments into the fund. The account pays 8% compounded annually. Find the amount of each payment. $381.83

53. To save for retirement, Karla Harby put $300 each month into an ordinary annuity for 20 years. Interest was compounded monthly. At the end of the 20 years, the annuity was worth $147,126. What annual interest rate did she receive? 6.3%

54. Jennifer Wall made payments of $250 per month at the end of each month to purchase a piece of property. After 30 years, she owned the property, which she sold for $330,000. What annual interest rate would she need to earn on an ordinary annuity for a comparable rate of return? 7.39%

55. When Joe and Sarah graduate from college, each expects to work a total of 45 years. Joe begins saving for retirement immediately. He plans to deposit $600 at the end of each quarter into an account paying 8.1% interest, compounded quarterly, for 10 years. He will then leave his balance in the account, earning the same interest rate, but make no further deposits for 35 years. Sarah plans to save nothing during the first 10 years and then begin depositing $600 at the end of each quarter in an account paying 8.1% interest, compounded quarterly, for 35 years.
(a) Without doing any calculations, predict which one will have the most in his or her retirement account after 45 years. Then test your prediction by answering the following questions (calculation required to the nearest dollar). Assume they both live to age 100. How much will Joe contribute to his retirement account? $279,000
(b) How much will Joe contribute to his retirement account? $279,000
(c) How much will Joe contribute to his retirement account? $279,000
(d) How much will Sarah contribute to her retirement account? $163,339
(e) How much will be in Sarah's account after 45 years? $163,339

56. In a 1992 Virginia lottery, the jackpot was $27 million. An Australian investment firm tried to buy all possible combinations of numbers, which would have cost $7 million. In fact, the firm ran out of time and was unable to buy all combinations, but ended up with the only winning ticket anyway. The firm received the jackpot in 20 equal annual payments of $1.35 million. Assume these payments meet the conditions of an ordinary annuity. (Data from: Washington Post, March 10, 1992, p. A1.)
(a) Suppose the firm can invest money at 8% interest compounded annually. How many years would it take until the investors would be further ahead than if they had simply invested the $7 million at the same rate? (Hint: Experiment with different values of n, the number of years, or use a graphing calculator to plot the value of both investments as a function of the number of years.) About 7 years
(b) How many years would it take in part (a) at an interest rate of 12%? About 8.6 years

Checkpoint Answers

1. (a) $2247.20 (b) $2120.00 (c) $6367.20
2. (a) $18,339.82 (b) $36,216.41
3. $872,354.36
4. (a) $232.38 (b) $262.97
5. (a) $104,812.44 (b) 8.9%
6. (a) $9446.24 (b) $9171.10
5.4 Exercises

Unless noted otherwise, all payments and withdrawals are made at the end of the period.

1. Explain the difference between the present value of an annuity and the future value of an annuity. Answer vary.

Find the present value of each ordinary annuity. (See Examples 1, 2, and 4.)

2. Payments of $890 each year for 16 years at 6% compounded annually $4094.25
3. Payments of $1400 each year for 8 years at 6% compounded annually $8093.71
4. Payments of $10,000 semiannually for 15 years at 7.5% compounded semiannually $198,192.45
5. Payments of $50,000 quarterly for 10 years at 5% compounded quarterly $1,556,346.85
6. Payments of $15,806 quarterly for 3 years at 6.8% compounded quarterly $719,255.47

Find the amount necessary to fund the given withdrawals. (See Examples 1 and 2.)

7. Quarterly withdrawals of $650 for 5 years; interest rate is 4.9%, compounded quarterly $41,448.10
8. Yearly withdrawals of $1200 for 14 years; interest rate is 5.6%, compounded annually $177,635.31
9. Monthly withdrawals of $425 for 10 years; interest rate is 6.1%, compounded monthly $18,188.61
10. Semiannual withdrawals of $3500 for 7 years; interest rate is 5.2%, compounded semiannually $496,636.21

Find the payment made by the ordinary annuity with the given present value. (See Example 3.)

11. $90,000; monthly payments for 22 years; interest rate is 4.9%, compounded monthly $557.68
12. $45,000; monthly payments for 11 years; interest rate is 5.3%, compounded monthly $330.52
13. $275,000; quarterly payments for 18 years; interest rate is 6%, compounded quarterly $6072.14
14. $330,000; quarterly payments for 30 years; interest rate is 6.1% compounded quarterly $6099.39

Find the lump sum deposited today that will yield the same total amount as payments of $10,000 at the end of each year for 15 years at each of the given interest rates. (See Example 4 and the box following it.)

15. 3% compounded annually $119,379.29
16. 4% compounded annually $111,183.87
17. 6% compounded annually $97,122.49
18. What sum deposited today at 5% compounded annually for 8 years will provide the same amount as $10000 deposited at the end of each year for 8 years at 6% compounded annually? $6699

19. What lump sum deposited today at 8% compounded quarterly for 10 years will yield the same final amount as deposits of $4000 at the end of each 6-month period for 10 years at 6% compounded semiannually? $48,572.34

Find the price a purchaser should be willing to pay for the given bond. Assume that the coupon interest is paid twice a year. (See Example 5.)

20. $20,000 bond with coupon rate 4.5% that matures in 8 years; current interest rate is 5.9% $18,234.70
21. $15,000 bond with coupon rate 6% that matures in 4 years; current interest rate is 5% $13,537.26
22. $25,000 bond with coupon rate 7% that matures in 10 years; current interest rate is 6% $26,590.68
23. $10,000 bond with coupon rate 5.4% that matures in 12 years; current interest rate is 6.5% $9955.14

24. What does it mean to amortize a loan? Answer vary.

Find the payment necessary to amortize each of the given loans. (See Examples 6, 7(a), and 8(a).)

25. $2500; 8% compounded quarterly; 6 quarterly payments $445.31
26. $41,000; 9% compounded semiannually; 10 semiannual payments $5181.53
27. $90,000; 7% compounded annually; 12 annual payments $71,272.32
28. $140,000; 12% compounded quarterly; 15 quarterly payments $111,272.32
29. $7400; 8.2% compounded semiannually; 18 semiannual payments $559.31
30. $5500; 9.5% compounded monthly; 24 monthly payments $352.33

Finance In April 2013, the mortgage interest rates listed in Exercises 31–34 for the given companies were listed at www.shsh.com. Find the monthly payment necessary to amortize the given loans. (See Example 7(a).)

31. $225,000 at 3.25% for 30 years from Amerisave $779.24
32. $330,000 at 3.125% for 20 years from Quicken Loans $1850.89
33. $140,000 at 2.375% for 15 years from Discover Home Loans $953.09
34. $180,000 at 2.25% for 10 years from Roundpoint Mortgage Company $1676.47

Finance Find the monthly payment and estimate the remaining balance (to the nearest dollar). Assume interest is on the unpaid balance. The interest rates are from national averages from www.bankrate.com in April 2013. (See Examples 7 and 8.)

35. Four-year new car loan for $26,799 at 3.13%; remaining balance after 2 years $19,97.25
36. Three-year used car loan for $15,875 at 2.96%; remaining balance after 1 year $14,613.38
37. Thirty-year mortgage for $210,000 at 3.54%; remaining balance after 12 years $147,69; $154,223.31
38. Fifteen-year mortgage for $195,000 at 2.78%; remaining balance after 4.5 years $132,610, $144,766.97

Use the amortization table in Example 8(c) to answer the questions in Exercises 39–42.

39. How much of the 5th payment is interest? $630
40. How much of the 10th payment is used to reduce the debt? $362.24
41. How much interest is paid in the first 5 months of the loan? $82.04
42. How much interest is paid in the last 5 months of the loan? $13.02

Find the cash value of the lottery jackpot (to the nearest dollar). Yearly jackpot payments begin immediately (26 for Mega Millions and 30 for Powerball). Assume the lottery can invest at the given interest rate. (See Example 9.)

43. Powerball: $57.6 million; 5.1% interest $104,660,929
44. Powerball: $207 million; 5.78% interest $201,877,031
45. Mega Millions: $41.6 million; 4.735% interest $22,761,636
46. Mega Millions: $23.4 million; 4.23% interest $14,624,398

Finance Work the following applied problems.

47. An auto stereo dealer sells a stereo system for $600 down and monthly payments of $30 for the next 3 years. If the interest rate is 1.25% per month on the unpaid balance, find
   (a) the cost of the stereo system; $1465.42
   (b) the total amount of interest paid. $214.58

48. John Kushida buys a used car costing $6000. He agrees to make payments at the end of each monthly period for 4 years. He pays 12% interest, compounded monthly.
   (a) What is the amount of each payment? $158.18
   (b) Find the total amount of interest Kushida will pay. $1384.86

Finance A student education loan has two repayment options. The standard plan repays the loan in 10 years with equal monthly payments. The extended plan allows from 12 to 30 years to repay the loan. A student borrows $35,000 at 7.43% compounded monthly.

51. Find the monthly payment and total interest paid under the standard plan. $414.78; $74,909.60
52. Find the monthly payment and total interest paid under the extended plan with 20 years to pay off the loan. $291.46; $27,340.40

Finance Use the formula for the approximate remaining balance to work each problem. (See Examples 7(b) and 8(b).)

53. When Teresa Flores opened her law office, she bought $14,000 worth of law books and $7200 worth of office furniture. She paid $1200 down and agreed to amortize the balance with semianual payments for 5 years at 12% compounded semiannually.
   (a) Find the amount of each payment. $2711.76
   (b) When her loan had been reduced below $5000, Flores received a large tax refund and decided to pay off the loan. How many payments were left at this time? 2

54. Karcem Adams buys a house for $285,000. He pays $60,000 down and takes out a mortgage at 6.9% on the balance. Find his monthly payment and the total amount of interest he will pay if the length of the mortgage is
   (a) 15 years; $2009.81; $136,765.80
   (b) 20 years; $1730.94; $490,035.60
   (c) 25 years; $1375.93; $3471,779
   (d) When will half the 20-year loan be paid off? After 199 payments (16.25 years)

55. Susan Carver will purchase a home for $257,000. She will use a down payment of 20% and finance the remaining portion at 3.9%, compounded monthly for 30 years.
   (a) What will be the monthly payment? $962.75
   (b) How much will remain on the loan after making payments for 5 years? $185,058.15
   (c) How much interest will be paid on the total amount of the loan over the course of 30 years? $143,516

56. Mohsen Manouchehri will purchase a $230,000 home with a 20-year mortgage. If he makes a down payment of 20% and the interest rate is 3.3%, compounded monthly,
   (a) what will the monthly payment be? $1048.34
   (b) how much will he owe after making payments to 8 years? $124,311.56
   (c) how much in total interest will he pay over the course of the 20-year loan? $67,594.40

Work each problem.

57. Elizabeth Bernardi and her employer contribute $400 at the end of each month to her retirement account, which earns 7% interest, compounded monthly. When she retires after 45 years, she plans to make monthly withdrawals for 30 years. If her account earns 5% interest, compounded monthly, then when she retires what is her maximum possible monthly withdrawal (withdrawing running out of money)? $8114.79
58. Jim Milliken won a $15,000 prize. On March 1, he deposited it in an account earning 5.2% interest, compounded monthly. On March 1 one year later, he begins to withdraw the same amount at the beginning of each month for a year. Assuming that he uses up all the money in the account, find the amount of each month's withdrawal. About $1353.95

59. Catherine Doanyos plans to retire in 20 years. She will make 20 years of monthly contributions to her retirement account. One month after her last contribution, she will begin the first of 10 years of withdrawals. She wants to withdraw $2500 per month. How large must her monthly contributions be in order to accomplish her goal if the account earns interest of 7.1% compounded monthly for the duration of her contributions and the 120 months of withdrawals? $406.53

60. David Turner plans to retire in 25 years. He will make 25 years of monthly contributions to his retirement account. One month after his last contribution, he will begin the first of 10 years of withdrawals. He wants to withdraw $3000 per month. How large must his monthly contributions be in order to accomplish his goal if the account earns interest of 6.8% compounded monthly for the duration of his contributions and the 120 months of withdrawals? $332.13

61. William Blake plans to retire in 20 years. William will make 10 years (120 months) of equal monthly payments into his account. Ten years after his last contribution, he will begin the first of 120 monthly withdrawals of $3400 per month. Assume that the retirement account earns interest of 8.2% compounded monthly for the duration of his contributions, the 10 years in between his contributions and the beginning of his withdrawals, and the 10 years of withdrawals. How large must William's monthly contributions be in order to accomplish his goal? $663.22

62. Gil Stevens plans to retire in 25 years. He will make 15 years (180 months) of equal monthly payments into his account. Ten years after his last contribution, he will begin the first of 120 monthly withdrawals of $2900 per month. Assume that the retirement account earns interest of 5.4% compounded monthly for the duration of his contributions, the 10 years in between his contributions and the beginning of his withdrawals, and the 10 years of withdrawals. How large must his monthly contributions be in order to accomplish his goal? $663.22

63. An insurance firm pays $4000 for a new printer for its computer. It amortizes the loan for the printer in 4 annual payments at 8% compounded annually.

64. Large semitrailer trucks cost $72,000 each. Ace Trucking buys such a truck and agrees to pay for it by a loan that will be amortized with 9 semiannual payments at 6% compounded semiannually.

65. One retailer charges $1048 for a certain computer. A firm of tax accountants buys 8 of these computers. It makes a down payment of $1200 and agrees to amortize the balance with monthly payments at 12% compounded monthly for 4 years.

66. Joan Varozza plans to borrow $20,000 to stock her small boutique. She will repay the loan with semiannual payments for 5 years at 7% compounded semiannually.

Checkpoint Answers
1. (a) $5989.07
   (b) $4968.20; $3865.66; $2674.91; $1388.90; $0.01
2. $20,602.37
3. $624.33
4. (a) $4832.97
   (b) $14,717.87
5. $10,817.57
6. $524.82
7. $98,605.61
8. $13,023,058.46

* indicates answer is in the Additional Instructor Answers at end of the book.