1. Calculate the following:

(a)  
\[
\lim_{x \to -1} \frac{x^2 - 4x - 5}{x + 1} = \lim_{x \to -1} \frac{(-5 + x)(1 + x)}{x + 1} = \lim_{x \to -1} (-5 + x) = -6
\]

(b)  
\[
\lim_{x \to 3} \frac{5x^2}{2x - 1} = \frac{5(3)^2}{2(3) - 1} = 9
\]

(c)  
\[
\lim_{x \to 2^+} \frac{4 - x^2}{|2 - x|} = \lim_{x \to 2^+} \frac{4 - x^2}{-(2 - x)} = \lim_{x \to 2^+} \frac{(2 - x)(x + 2)}{(2 - x)} = \lim_{x \to 2^+} -(x + 2) = -4
\]

(d)  
\[
\lim_{x \to 1} \sqrt{\frac{2x^3 - 3x + 5}{2 - x}} = \sqrt{\frac{2 \cdot 1^3 - 3 \cdot 1 + 5}{2 - 1}} = \sqrt{4} = 2
\]

(e)  
\[
\lim_{x \to 3} f(x) \text{ where } f(x) = \begin{cases} 
  x^2 & \text{if } x > 3 \\
  8 & \text{if } x = 3 \\
  12 - x & \text{if } x < 3
\end{cases}
\]

Notice that \(\lim_{x \to 3^-} f(x) = \lim_{x \to 3^-} (12 - x) = 9\) and \(\lim_{x \to 3^+} f(x) = \lim_{x \to 3^+} x^2 = 9\). Since both one-sided limits agree with the value 9, \(\lim_{x \to 3} f(x) = 9\).

(f)  
\[
\lim_{x \to 3^-} \frac{x + 3}{x^2 - 9} = \lim_{x \to 3^-} \frac{x + 3}{(x - 3)(x + 3)} = \lim_{x \to 3^-} \frac{1}{x - 3} = -\infty
\]

(g) The horizontal and vertical asymptotes of  
\[
y = \frac{4 - 3x}{\sqrt{16x^2 + 1}}
\]

Since \(\lim_{x \to -\infty} \frac{4 - 3x}{\sqrt{16x^2 + 1}} = -\frac{3}{4}\) and \(\lim_{x \to -\infty} \frac{4 - 3x}{\sqrt{16x^2 + 1}} = \frac{3}{4}\), the horizontal asymptotes are \(y = \frac{3}{4}\) and \(y = -\frac{3}{4}\). There are no vertical asymptotes since \(\frac{4 - 3x}{\sqrt{16x^2 + 1}}\) exists for all \(x\) values.

(h)  
\[
f'(x) \text{ where } f(x) = \sin \left( x^{100} \right)
\]

Applying the chain rule, we obtain  
\[
f'(x) = 100x^{99} \cos \left( x^{100} \right)
\]
(i) \( f'(x) \) where
\[ f(x) = \sqrt{e^{2x} + 7x} \]
Applying the chain rule, we obtain
\[ f'(x) = \frac{1}{2}(e^{2x} + 7x)^{-\frac{1}{2}} (2e^{2x} + 7). \]

(j) \( f'(x) \) where
\[ f(x) = 10^\cos x \]
Rewrite \( 10^\cos x = e^{\cos x \ln 10} \) then applying the chain rule, we obtain
\[ f'(x) = e^{\cos x \ln 10} (-\sin x \ln 10) = -(\sin x \ln 10)10^\cos x. \]

(k) \( f'(x) \) where
\[ f(x) = \frac{\ln x}{x} \]
Use the quotient rule to obtain
\[ f'(x) = \frac{x \frac{1}{x} - \ln x}{x^2} = \frac{1 - \ln x}{x^2} \]

(l) \( f'(x) \) where
\[ f(x) = x^2 \]
First, rewrite as \( f(x) = \exp(x^2 \ln x) \) then
\[ f'(x) = \exp(x^2 \ln x) \left( 2x \ln x + x^2 \frac{1}{x} \right) = x^2 (2x \ln x + x) \]

(m) \( f'(x) \) where
\[ f(x) = \arctan(x^3) \]
\[ f'(x) = \frac{1}{1 + (x^3)^2} 3x^2 = \frac{3x^2}{1 + x^6} \]

2. Consider the function
\[ f(x) = \begin{cases} x - c, & \text{if } x > 2; \\ 3x^2, & \text{if } x \leq 2 \end{cases} \]
where \( c \) is a real number.

(a) What value of \( c \) makes the function \( f \) continuous everywhere? The only place where the function \( f \) might not be continuous is at \( x = 2 \) but we can avoid this if we choose \( c \) so that the graphs of \( y = x - c \) and \( y = 3x^2 \) agree at \( x = 2 \), i.e. so there is no jump across \( x = 2 \). Solving \( 2 - c = 3 \cdot 2^2 \) means that \( c = -10 \).

(b) If \( x > 2 \) then \( f(x) = x - c \Rightarrow f'(x) = 1 \Rightarrow f'(7) = 1 \)

(c) If \( x < 2 \) then \( f(x) = 3x^2 \Rightarrow f'(x) = 6x \Rightarrow f'(-1) = -6 \)
Call \( f_1(x) = x - c \) and \( f_2(x) = 3x^2 \). Notice that \( f'(2) \) does not exist (even when \( c = -10 \)) since \( f'_1(2) = 1 \) which does not agree with \( f'_2(2) = 6 \cdot 2 = 12 \).

3. Compute the derivative of the following functions:
   
   (a) \( f(x) = \pi^4 \Rightarrow f'(x) = 0 \)
   
   (b) \( f(x) = 3x^5 - x^2 + 9 \Rightarrow f'(x) = 15x^4 - 2x \)
   
   (c) \( f(x) = \frac{2}{x^2} - 3\sqrt{x} = 2x^{-2} - 3x^{1/2} \Rightarrow f'(x) = -4x^{-3} - \frac{3}{2}x^{-\frac{1}{2}} \)
   
   (d) \[
   f(x) = \frac{x^2}{2x - 3} \Rightarrow \]
   \[
   f'(x) = \frac{(2x - 3) \frac{d}{dx}(x^2) - x^2 \frac{d}{dx}(2x - 3)}{(2x - 3)^2} = \frac{(2x - 3)(2x) - x^2(2)}{(2x - 3)^2} = \frac{2x^2 - 6x}{(2x - 3)^2}
   
   (e) \( f(x) = x^2e^x \Rightarrow f'(x) = e^x \frac{d}{dx}(x^2) + x^2 \frac{d}{dx}(e^x) = 2xe^x + x^2e^x \)
   
   (f) \( f(t) = t \sin t \Rightarrow f'(t) = t \frac{d}{dt}(\sin t) + \sin t \frac{d}{dt}(t) = t \cos t + \sin t \)

4. Find the equation for the tangent line to the curve \( y = f(x) \) where through the point \((1, -3)\) where \( f(x) = x^8 - 4x \). \( f'(x) = 8x^7 - 4 \). The slope of the tangent line is \( f'(1) = 4 \). Therefore the equation for this tangent line is \[
\frac{y - (-3)}{x - 1} = 4
\]
or, solving for \( y \), \( y = 4x - 7 \).

5. Suppose an object is moving along the real line with its position at time \( t \) given by the function \( s(t) = \frac{1}{3}t^3 - 3t^2 - 7t + 10 \).
   
   (a) The object is at rest when the velocity \( v(t) = 0 \) but \( v(t) = s'(t) = t^2 - 6t - 7 = (t - 7)(t + 1) = 0 \). The solution is \( t = -1 \) or \( t = 7 \).
   
   (b) The object is decelerating when the acceleration \( a(t) < 0 \) but \( a(t) = v'(t) = 2t - 6 < 0 \) when \( t < 3 \).
(c) \( v(2) = -15 \).

6. Find the equation for the tangent line to the curve given by the equation \( \cos(xy) - 3y^3 = e^x + 1 \) through the point \((0, -1)\).

We need to find \( \frac{dy}{dx} \). Use implicit differentiation to obtain

\[
\frac{d}{dx} (\cos(xy) - 3y^3) = \frac{d}{dx} (e^x + 1)
\]

which yields

\[
-\sin(xy)(xy' + y) - 9y^2y' = e^x
\]

Now, solve to obtain

\[
\frac{dy}{dx} = \frac{-e^x + y\sin(xy)}{9y^2 + x\sin(xy)}
\]

At the point \((x, y) = (0, -1)\), \( \frac{dy}{dx} = -\frac{1}{9} \) so the equation for the tangent line is

\[
\frac{y - (-1)}{x - 0} = -\frac{1}{9}
\]

or \( y = -1 - \frac{1}{9}x \).

7. Consider the graph of \( y = f(x) \) on the next page (figure 1).

(a) Where is \( f \) undefined?

\( x = -1, 0, 3 \)

(b) Where is \( f \) not continuous?

\( x = -5, -1, 0, 3 \)

(c) Where is \( f \) not differentiable?

\( x = -5, -3, -2, -1, 0, 3, 5 \)

(d) On what interval(s) is \( f' \) positive? Where does \( f' \) vanish? \( f' \) is positive on the intervals \((-5, -4)\), \((-2, -1)\), \((2, 3)\), \((3, 5)\). \( f' \) vanishes at \( x = -4, 2 \)

(e) On what interval(s) is \( f \) concave down? \((-\infty, -5), (-5, -3), (-1, 0), (3, 5)\)

(f) What are

i. \( f'(6) = \frac{0 - 3}{7.25 - 5} = -\frac{3}{2.25} \)

ii. \( \lim_{x \to -5^+} f(x) = -2 \)

iii. \( \lim_{x \to -1} f(x) = -1 \)

iv. \( \lim_{x \to -\infty} f(x) = 0 \)