Math 123, Practice Exam #2, October 29, 1999

1. Find the following:
   (a) 
   \[ \lim_{y \to \infty} \left( 1 + \frac{2}{y} \right)^y \]
   (b) 
   \[ \lim_{x \to 0} \frac{\cos(\sqrt{5}x) - 1}{x^2} \]
   (c) The horizontal and vertical asymptotes of 
   \[ y = \frac{4 - 3x}{\sqrt{16x^2 + 1}} \]
   (d) \( f'(x) \) where 
   \[ f(x) = \sin(x^{100}) \]
   (e) \( f'(t) \) where 
   \[ f(t) = \tan(\sqrt{t^4 + 2}) \]
   (f) \( f'(x) \) where 
   \[ f(x) = \frac{\ln x}{x} \]
   (g) \( f'(x) \) where 
   \[ f(x) = x^2 \]
   (h) \( f'(x) \) where 
   \[ f(x) = 10\cos x \]
   (i) \( f'(x) \) where 
   \[ f(x) = \arctan(x^3) \]

2. Find the equation for the tangent line to the curve given by the equation 
   \( \cos(xy) - 3y^3 = e^x + 1 \)
   through the point \((0, -1)\).

3. We wish to find an approximate value of the positive root of 
   \( 2 \sin x - x = 0 \) using Newton’s Method.
   (a) Find the formula for \( x_{n+1} \) in terms of \( x_n \).
   (b) Find the positive root (up to 5 decimal places) of this equation 
       using Newton’s method using the initial value \( x_1 = 1 \). Make a table 
       of all \( x_n \) values which you need to produce your answer.

4. A man 6 feet tall is walking away from a light pole which is 30 feet high. If the tip of his shadow 
   is moving at a rate equal to the distance between him and the light pole (in feet) then how fast 
   is the man walking when he is 24 feet from the pole?

5. A spherical snowball is melting at a rate equal to its surface area. How fast is its radius shrinking 
   when its volume is equal to its surface area?

6. Solve the following:
   (a) Find the area of the largest rectangle that can be inscribed in a semicircle of radius \( r \).
(b) Two nonnegative numbers are such that the sum of the first number and 3 times the second number equals 10. Find these numbers if the sum of their squares is as small as possible.

7. Consider the function

\[ f(x) = 1 + x - 3x^{\frac{2}{3}} \]

(a) Find the critical point(s) of \( f \).
(b) On what interval(s) is \( f \) increasing?
(c) On what interval(s) is \( f \) concave down?
(d) Find the inflection point(s) of \( f \).
(e) Find the local minimum (minima) of \( f \).
(f) Find the global maximum of \( f \) on the interval \([-2, 3]\)

8. Suppose the graph on the next page is \( y = f(x) \).

(a) Find the interval(s) where \( f \) is increasing.
(b) Find the interval(s) where \( f \) is concave up.
(c) Find the inflection point(s) of \( f \).
(d) Find the critical numbers of \( f \).
(e) Find the local maximum (maxima) of \( f \).
(f) Find the horizontal asymptotes of \( f \).

9. Suppose the graph on the next page is \( y = f'(x) \) where we assume that \( f \) is continuous everywhere.

(a) Find the interval(s) where \( f \) is increasing.
(b) Find the interval(s) where \( f \) is concave up.
(c) Find the inflection point(s) of \( f \).
(d) Find the critical numbers of \( f \).
(e) Find the local maximum (maxima) of \( f \).
(f) Find \( f''(7) \).