1. Find the following:

(a) \[ \int \sin^{100}(x) \cos(x) \, dx \]

(b) \[ \int \frac{3x^2 - 16x + 5}{\sqrt{x^3 - 8x^2 + 5x + 3}} \, dx \]

(c) \[ \int (x - \frac{3}{2}) \sin(x^2 - 3x) \, dx \]

(d) \[ \int x \ln x^3 \, dx \]

(e) \[ \int e^{\sqrt{x}} \, dx \]

(f) \[ \int_{0}^{2} \tan^5 x \sec^2 x \, dx \]

(g) \[ \int_{2}^{4} f'(x) \sin(f(x)) \, dx \]

if \( f \) is a continuous function on the interval \([0, 20]\) such that \( f(0) = 3, f(2) = 1, f(4) = 7, \) and \( f(20) = 5. \)

(h) \[ \int_{1}^{3} (2x - 8) e^{-x} \, dx \]

(i) \[ \int_{-\infty}^{3} \frac{1}{1 + x^2} \, dx \]

(j) \( \bar{f}, \) the average value of \( f(x) = x^2 \) over the interval \([3, 8]. \)

2. Consider the region \( R \) in the xy-plane where \( x \geq 0 \) bounded by the graphs \( y = x^3 \) and \( y = x^5. \)

(a) Calculate the area of \( R. \)

(b) Calculate the centroid of \( R. \)

(c) Consider the solid \( S \) whose base is \( R \) and whose cross-sections perpendicular to the \( x \)-axis are equilateral triangles. Find the volume \( V \) of \( S. \)
3. Find the arc length of the part of the curve \( y = 2x^2 \) between the points \((1, 2)\) and \((4, 16)\).

4. Consider the region \( Q \) in the xy-plane bounded by the graphs \( y = x \) and \( y = (x - 2)^2 \). Find the volume of the solid obtained by revolving \( Q \) about the x-axis.

5. Consider the differential equation

\[
\frac{dy}{dx} + 2y = e^x.
\]

(a) Verify that

\[
y = Ce^{-2x} + \frac{1}{3}e^x
\]

is a solution for any constant \( C \).

(b) Find the solution which satisfies \( y(0) = 8 \).