Problem 1 (10 points) Find $c$ such that $P(X = k) = c\lambda^k/k!, \; k = 2, 3, \ldots, \lambda > 0$ is a PMF.

Problem 2 (10 points) A point moves along a line to the left and the right with probability $p$ and $1 - p$, respectively, at each unit of time. It starts from the origin at time 0. Let $X_n$ be the number of left moves until time $n$ and $Y_n$ be the position of the point at time $n$. Find the PMF of $X_n$ and $Y_n$.

Problem 3 (15 points) In your local state lottery, a drawing is held each week where 5 numbers between 1 and 37 are chosen randomly, without replacement and their order does not matter. Before the drawing, you can purchase a ticket on which you guess 5 numbers (also between 1 and 37, without replacement, order does not matter).

1. What is the probability of guessing exactly 3 numbers from the drawing correctly?
2. Let $X$ be the number of people among $10^6$ people guessing all 5 numbers correctly. Identify the distribution of $X$.
3. What is the probability of 2 or more people among $10^6$ people guessing all 5 numbers correctly?

Problem 4 (15 points) Draw two numbers at random from $\{1, 2, 3, 4\}$ without replacement. Let $X$ denote the smaller of the two numbers and $Y$ denote the larger of the two numbers.

1. Make a table which contains the joint probability mass function $p_{X,Y}(x,y)$ of $X$ and $Y$ as well as the marginal distributions of $X$ and $Y$.
2. Use your result from part (1) to compute $P(X \leq 2 | Y = 4)$.

Problem 5 (10 points) A health study tracked a group of people for five years. At the beginning of the study, 20% were classified as heavy smokers, 30% as light smokers, and 50% as nonsmokers.

Results of the study showed that light smokers were twice as likely as nonsmokers to die during the five-year study, but only half as likely as heavy smokers.

A randomly selected participant from the study died over the five-year period. Calculate the probability that the participant was a heavy smoker.
Problem 6 (10 points) An insurance company examines its pool of auto insurance customers and gathers the following information:

1. All customers insure at least one car.
2. 70% of the customers insure more than one car.
3. 20% of the customers insure a sports car.
4. Of those customers who insure more than one car 15% insure a sports car.

Calculate the probability that a randomly selected customer insures exactly one car and that car is not a sports car.

Problem 7 (20 points) Let X and Y be independent random variables, both having the geometric distribution with parameter p. Determine and identify the PMF of the random variable

1. X + Y
2. min\{X,Y\}

(Extra Credit 10 points) Amy enjoys cooking and eating spaghetti. One day, she dropped a piece of uncooked spaghetti of length 1 and found out that it broke into three smaller pieces. What is the probability that the three pieces happen to form a triangle. (Recall that the lengths of any two sides of a triangle must add to more than the length of the third side.)