**Suggestions, P.S. 9**

8.6.

**Using smoothing splines:**

(a) Fitting a smoothing spline (as in section 5.4) can be done with various computer packages. The R package smooth.spline is a good choice. For confidence bands note you can estimate variance of the fitted function \( \hat{f} \) at the data points using \( \hat{f} = S_\lambda y \), with \( \hat{f} = \hat{y} \) the vector of fitted values \( \hat{f}(x_i) \). Show 

\[
V(\hat{f}) = S_\lambda V(y) S_\lambda^T,
\]

with \( V(\cdot) \) the covariance matrix. This gives variance of \( f(x_i) \); you can interpolate this (even with splines) to get the variance of \( f(x) \). Note you can get a column of \( S_\lambda \) by letting \( y \) above be the vector with all zeroes and a single 1, using a package like smooth.spline.

(b) Here the matrix \( H \) will be \( N \times p \), where in this case \( p \) is the number of basis functions \( h_j(x) \), which for smoothing splines is the set of *natural* splines with knots at all of the data points \( x_i \). Again you can try a medium \( (\tau = 1) \) and a large value of \( \tau \).

(c) Here the smoothing splines are computed on bootstrap samples; overlapping sample points should not cause any problems.