MA 412 COMPLEX ANALYSIS

ANALYTIC AND HARMONIC FUNCTIONS

Section 1.6: 9,10,11,13.

Section 2.1: 6,7,8,9.

Ex. 1: Using Thm II.3 and properties of limits, show that the limit of a polynomial \( P(z) = a_0 + a_1 z + a_2 z^2 + \cdots + a_n z^n \) as \( z \) approaches \( z_0 \), is given by \( P(z_0) \). That is, show

\[
\lim_{z \to z_0} P(z) = P(z_0).
\]

(Hint: do not use \( \epsilon \) and \( \delta \) arguments.)

Ex. 2: Verify Thm II.6.

Ex. 3: Verify Thm II.7.

Section 2.5: 1,2,6,7,8,11,12,18,21.

Ex. 4: Show that

1. \( f(z) = \overline{z} \) is continuous at every point in \( \mathbb{C} \).
2. \( f \) is nowhere differentiable.
3. If the function \( g \) is differentiable at \( z_0 \), then \( g \) is continuous at \( z_0 \).

Ex. 5: Using the differentiation formulas, show that

1. a polynomial \( P(z) = a_0 + a_1 z + \cdots + a_n z^n \), \( (a_n \neq 0) \) of degree \( n \geq 1 \) is differentiable everywhere, with derivative

\[
P'(z) = a_1 + 2a_2 z + \cdots + na_n z^{n-1}.
\]

2. the coefficients in the polynomial \( P(z) \) in part 1 can be written as

\[
a_k = P^{(k)}(0)/k!
\]

for \( k = 0, 1, \ldots, n \).

Section 3.1: 1,6,9,11,13.

Ex. 6: If the Cauchy-Riemann equations in polar form hold at \( (x_0, y_0) \), show that the Cauchy-Riemann equation hold in cartesian coordinates at the same point.
(Hint: solve equations

\[ u_r = u_x \cos \theta + u_y \sin \theta \]
\[ u_\theta = -u_x r \sin \theta + u_y r \cos \theta \]

and those similar for \( v_r \) and \( v_\theta \). Then use the expressions obtained in the polar form of the CR eqn's.)

**Section 3.2**: 1,7,9,18,19.

**Section 3.3**: 1,5,10