Problem 1 During the 2009 regular season, the Red Sox played 162 games, and had a record of 95 wins and 67 loses. This was good enough to put them in the playoffs, where they are now playing to win the World Series. For the Red Sox, the playoffs consist of 3 rounds – The ALDS, the ALCS, and finally the World Series. If they lose one round, they are out of the playoffs.

a) Based on their regular season win-loss record, and assuming each game was independent, estimate the probability $p$ that the Red Sox win any given baseball game. (i.e. compute their winning-percentage in 2009)

b) The first round of the playoffs is a best-of-5 game series (meaning the Red Sox play up to 5 games and move on if they win 3 of these games). Assuming the winning-percentage $p$ you computed in part a) continues to hold in the playoffs, and that each game is independent of the others, what is the probability $p_5$ that the Red Sox win a best-of-5 game series? (you may give your answer in terms of $p$ if you do not have a calculator)

c) The second and third rounds are best-of-7 game series (meaning they have to win 4 of these games to win the series). What is the probability $p_7$ that the Red Sox win a best-of-7 game series? (again, you may give your answer in terms of $p$)

d) Assuming these three rounds in the playoffs are independent, what is the probability the Red Sox win the World Series? (i.e. that they win one best-of-5 series, and two best-of-7 series).

Remark: This was a very over-simplified model. If you go online and check what professional odds-makers say the Sox’s chances are, they are much lower than what we got here. For the baseball fans among you, can you think of ways to improve this model?

Problem 2 (more baseball) The Chicago Cubs think that they are cursed (or at least some of their fans do). This team has gone 101 years without winning a World Series. Assuming that this team has remained more-or-less average over the years, we can say that for any given year, their probability of winning the World Series is $p = 1/30$ (since there are 30 teams in Major League Baseball). Assuming this, what is the probability that the Cubs go more than 101 years without winning a World Series? Based on this, do you think they are cursed?

Problem 3 I place two urns in front of you which contain 10 colored balls each. I tell you that one contains 7 red balls and 3 blue balls and the other contains 5 red balls and 5 blue balls. You pick an urn at random and draw 2 balls (without replacement). One is red and one is blue. What is the probability you chose the urn with 5 red balls and 5 blue balls?

Problem 4 a) Say $X \sim Bin(n_1, p)$ and $Y \sim Bernoulli(p)$ are independent. What is the distribution of $X + Y$? Explain why.

b) Say $X \sim Pascal(r_1, p)$ and $Y \sim Geometric(p)$ are independent. What is the distribution of $X + Y$? Explain why.