MA 230 Problem of the Day February 3, 2003

Let 

\[ f(x, y) = y \cos x. \]

Find the points on the graph of \( f \) where the tangent plane is parallel to the plane

\[ x - \sqrt{3} y + 2z = -2. \]

\[ \frac{\partial f}{\partial x} = -y \sin x \quad \frac{\partial f}{\partial y} = \cos x \]

Normal vector to the tangent plane is 

\[ \vec{T} = (-y \sin x) \hat{x} + (\cos x) \hat{y} - \hat{k} \]

Normal vector to plane is 

\[ \vec{N} = \hat{x} - \sqrt{3} \hat{y} + 2\hat{k} \]

The planes are parallel if \( \vec{T} \) and \( \vec{N} \) are parallel, i.e., \( \vec{T} = \lambda \vec{N} \) for some scalar \( \lambda \). We have

\[
\begin{cases}
- y \sin x = \lambda \\
\cos x = -\sqrt{3} \lambda \\
- 1 = 2 \lambda 
\end{cases}
\]

\[ \Rightarrow \lambda = -\frac{1}{2} \Rightarrow \cos x = \frac{\sqrt{3}}{2} \Rightarrow \sin x = \mp \frac{1}{2}. \]

We have \( x = \pm \frac{\pi}{6} + 2k\pi \). If \( x = \frac{\pi}{6} + 2k\pi \), then \( y = 1 \), and the point is \( (\frac{\pi}{6} + 2k\pi, 1, \frac{\sqrt{3}}{2}) \). If \( x = -\frac{\pi}{6} + 2k\pi \), then \( y = -1 \) and the point is \( (\frac{\pi}{6} + 2k\pi, -1, -\frac{\sqrt{3}}{2}) \).