Absolute Values and Inequalities

One of the ways to get good at manipulating absolute values and inequalities is to do complicated problems, like “For which \( x \)'s is

\[
\frac{|x^2 - 3x - 2|(x - 3)}{| - x^2 + 3|} > 0?
\]

But most problems you will encounter are not so complicated.

One of the things you must train yourself to do is not to work too hard on the easy problems. Whenever you start a problem, the first question should not be “How many cases will I have to do?” or “What is the general technique for this type of problem?” The first question you should always ask is Is this problem easy?

If the answer is “no” or “I don’t know if it is easy”, then start thinking about techniques. If the answer is “yes” then just do it.

For example:

1. For what \( x \) values is

\[
|x^2 + 8x - 7| \geq 0?
\]

Since the right hand side is entirely inside the absolute value, the right hand side will ALWAYS be \( \geq 0 \). We don’t have to compute anything, the inequality holds for all \( x \) values.

2. For what \( x \) values is

\[
|x - 3| > 1
\]

Remember that absolute value is a way of measuring length. That is \( |x - 3| \) is the distance from 3 to \( x \). So the question is asking what are the numbers that are more than one from 3. So the solution set is \( x > 4 \) and \( x < 2 \) (or the interval \((-\infty, 2) \) union \((4, \infty))\).

3. For what \( x \) values is

\[
|2x - 8| \leq 3
\]

If we divide both sides by 2, this inequality is

\[
\frac{|2x - 8|}{2} \leq \frac{3}{2}
\]

or

\[
|x - 4| \leq \frac{3}{2}.
\]
So, as above, the problem is asking which values of $x$ are within $3/2$ of $4$. The solution set is

$$4 - \frac{3}{2} \leq x \leq 4 + \frac{3}{2}$$

or

$$\frac{5}{2} \leq x \leq \frac{11}{2}.$$  

4. For what values of $x$ is

$$x^2 + 4x + 4 > 0?$$

Well we remember (or will as soon as we review polynomials) that $x^2 + 4x + 4 = (x + 2)^2$, and so the problem is for which $x$ is

$$(x + 2)^2 > 0?$$

The solution is all $x \neq -2$ or $(-\infty, -2)$ union $(-2, \infty)$.

So how can you tell if a problem is easy? The only ways are experience and remembering to look first. If you do it the hard way and get the right answer, it is ok, it just takes longer (and is a little embarrassing).