1. Farmer Jones has 100 feet of fence. She wants to make a rectangular pen using all the fence for three sides, and the side of the barn for the fourth side. What is the area as a function of the length.

**Solution:** Let $L =$ length of pen (in feet) and $W =$ width (in feet).

Then the area $A$ is

$$A = LW.$$  

To express this area as a function of the length, we note that $2W + L = 100$  
(Since this is how much fence she has)

So

$$2W = 100 - L,$$

or

$$W = 50 - \frac{L}{2}.$$  

So the area is (replacing $W$ by $50 - \frac{L}{2}$)

$$A(L) = L \cdot (50 - \frac{L}{2})$$

or

$$A(L) = 50L - \frac{L^2}{2}.$$  

Either is OK.
2. A piece of wire 2 meters long is cut into 2 pieces.

One piece is made into a square, the other is made into a circle.

What is the area enclosed?

**Answer:** It depends on how you cut the wire.

Let $x = \text{length of piece made into circle}$ so $2 - x = \text{length of piece made into square}.$

Side length of square is $\frac{2-x}{4}$.

So the area of the square is \( \left(\frac{2-x}{4}\right)^2 \).

The circumference of the circle is $x$ (the length of wire).

Since circumference $= 2\pi \times \text{radius}$, the radius of the circle is $\frac{x}{2\pi}$ (i.e., $2\pi \times \text{radius} = x$).

So radius $= \frac{x}{2\pi}$.

So the area is $\pi \left(\frac{x}{2\pi}\right)^2 = \pi \frac{x^2}{4\pi^2} = \frac{x^2}{4\pi}$.

So the total area as a function of $x$ is

\[ A(x) = \left(\frac{2-x}{4}\right)^2 + \frac{x^2}{4\pi}. \]

(The answer is different if you make the $x$ length into square and $2-x$ into circle. It is

\[ A(x) = \left(\frac{2-x}{4}\right)^2 + \frac{(2-x)^2}{4\pi} \text{ area}. \]
3. You cut a 3 meter piece of wire into two pieces. One is a square and one is an equilateral triangle. What is the total area enclosed.

Answer: Let \( x \) = length made into triangle so \( 3 - x \) = length made into square.

Area of square is \( \left(\frac{3-x}{4}\right)^2 \)

For the equilateral triangle

Each side is length \( \frac{\sqrt{3}}{3} \)

so the height is \( \frac{\sqrt{3}}{2} \cdot \frac{x}{3} \)

(This is a fact about equilateral triangles)

so the area is \( \frac{\sqrt{3}}{2} \cdot \frac{x}{3} \cdot \frac{\sqrt{3}x}{2} \)

\( = \frac{x^2}{36} \)

So the total area is

\( A(x) = \left(\frac{3-x}{4}\right)^2 + \frac{\sqrt{3}x^2}{36} \)

The height of the equilateral triangle you can remember the "standard" size

\[ \frac{1}{2} \]