MA 124 Midterm 1

Clearly print your NAME ____________________________

                      Last (Family) Name,                      First (Given) Name

Clearly write your BU ID NUMBER ____________________________

I understand that my conduct during this exam is governed by the BU Academic Conduct Code which requires that my work be entirely my own.

Signature ____________________________

Circle Your Discussion Section time
B1 Wed. 12:00-1:30       B2 Tue. 12:30-2:00       B3 Tue. 3:30-5:00       B4 Mon. 4-5:30
B5 Thur. 12:30-2:00      B6 Thur. 3:30-5:00       B6 Wed. 1:30-3:00

NOTE: If you take the exam from the wrong section, up to 10 points will be deducted from your score.

<table>
<thead>
<tr>
<th>Problem</th>
<th>Points</th>
<th>Score</th>
</tr>
</thead>
<tbody>
<tr>
<td>Name/Section</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>Problem 1</td>
<td>21</td>
<td></td>
</tr>
<tr>
<td>Problem 2</td>
<td>15</td>
<td></td>
</tr>
<tr>
<td>Problem 3</td>
<td>15</td>
<td></td>
</tr>
<tr>
<td>Problem 4</td>
<td>16</td>
<td></td>
</tr>
<tr>
<td>Problem 5</td>
<td>15</td>
<td></td>
</tr>
<tr>
<td>Problem 6</td>
<td>20</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>104</td>
<td></td>
</tr>
</tbody>
</table>
INSTRUCTIONS FOR ALL PROBLEMS: Show your work. Neatness definitely counts.

1. Compute the following:

(a) \[ \int 3x \ln(x) \, dx = \int 3x \ln(x) \, dx = 3 \left( \frac{x^2}{2} \ln(x) - \frac{x^2}{2} \right) \, dx \]

\[ \text{Parts} \quad u = \ln(x), \quad \frac{du}{dx} = \frac{1}{x} \quad v = \frac{x^2}{2} \]

\[ \frac{du}{dx} = \frac{1}{x} \quad \Rightarrow \quad u = \ln(x) \quad \frac{dv}{dx} = \frac{x^2}{2} \]

\[ = 3 \left( \frac{x^2}{2} \ln(x) - \frac{3}{2} \int x \, dx \right) \]

\[ = 3 \left( \frac{x^2}{2} \ln(x) - \frac{3x^2}{4} + C \right) \]

(b) \[ \int_0^2 xe^{x^2+1} \, dx = \int \sqrt{u} \, du = \frac{1}{6} \left( e^{u^3} - e^1 \right) \]

Let \( u = 3x^2 + 1 \)

\[ \frac{du}{dx} = 6x \quad \Rightarrow \quad \frac{1}{6} \, du = x \, dx \]

\[ u = 0 \Rightarrow u = 1, \quad u = 2 \Rightarrow u = 13 \]

(c) \[ \int \frac{x+7}{\sqrt{x^2+2}} \, dx = \int \frac{u}{\sqrt{u}} \, du = \int \frac{u^{1/2}}{\sqrt{u}} \, du \]

Let \( u = x+2 \)

\[ \frac{du}{dx} = 1 \Rightarrow \, dx = du \quad u = 2 \]

\[ = \int u^{1/2} + 5u^{-1/2} \, du \]

\[ = \frac{u^{3/2}}{3/2} + 5 \frac{u^{1/2}}{1/2} + C \]

\[ = \frac{2}{3} \sqrt{x+2}^3 + 10 \sqrt{x+2} + C \]
2. A solid object has base given by the region in the $xy$-plane between $y = 0$ and $y = 4-x^2$ with $y \geq 0$, and has cross sections perpendicular to the $x$-axis that are semi-circles.

Set up, BUT DO NOT EVALUATE, the integral for the volume of this solid.

Slice perpendicular to the $x$ axis width $\Delta x$.

The slice at $x$ is a half cylinder with radius $\frac{1}{2}(4-x^2)$ and length $\Delta x$.

So volume

\[
\frac{1}{2} \pi \left(\frac{4-x^2}{2}\right)^2 \Delta x
\]

Adding and taking the limit as $\Delta x \to 0$ gives

\[
\int_{-2}^{2} \frac{\pi}{8}(4-x^2)^2 \, dx
\]
3. A solid is formed by revolving the region bounded by $x = 0$, $y = 3$, and $y = \sqrt{x+4}$ about the $y$-axis.

Set up, BUT DO NOT EVALUATE, the integral for the volume of this solid.

Slice into disks perpendicular to the $y$-axis.

Radius: Where

$\sqrt{x+4} = y$, $x+4 = y^2$

$x = y^2 - 4$

Volume disk at $y$ is $\pi (y^2 - 4)^2 \Delta y$

So limit $\Delta y \to 0$ gives

$$\int_{2}^{3} \pi (y^2 - 4)^2 \, dy$$
4. Suppose a 10 meter chain is hanging from the side of a 100 meter tall building. Suppose the upper 5 meters (the part closer to the top of the building) is very light carbon fiber with density only 0.1 kg/meter, but the bottom 5 meters (the part closer to the ground) is metal with a density of 2 kg/meter.

(a) Compute the work necessary to pull the entire chain to the top of the building. (For convenience, you may assume the acceleration of gravity is 10 meters/sec².)

\[ z = \text{distance from top} \]

Slice chain into pieces size \(\Delta z\) in \(0 < z < 5\) piece is lifted \(z\) m

\[
\text{Force} = \text{mass} \times \text{accel} = 0.1 \times \Delta z \text{ m} \times 10 \text{ m/sec}^2
\]

\[
F = \frac{\Delta z \text{ kg m/sec}^2}{2}
\]

So work \(\int_0^5 zdz = \frac{z^2}{2}\bigg|_0^5 = \frac{25}{2} \text{ kg m}^2\text{/sec}^2\)

For \(5 < z < 10\) same except mass \(\Delta z\)

So work \(\int_5^{10}zdz = 102 \bigg|_5^0 = 1000 - 250 = 750 \text{ kg m}^2\text{/sec}^2\)

Total work \(750 + \frac{25}{2} = 767.5 \text{ kg m}^2\text{/sec}^2\)

(b) Suppose the chain was turned around so the carbon fiber section was closer to the ground and the metal closer to the top of the building. Would the work raising the chain to the top of the building be more or less? Explain your answer.

Less because the more massive part of the chain is moved up less distance.
5. Suppose you are given the velocity in meters per second of a particle moving along a line in the form of the graph below. What is the total distance travelled by the particle from time \( t = 0 \) to \( t = 5 \) (Show your work so that we can tell how you computed your answer.)

\[
\int_{0}^{5} \text{velocity} \, dt = \text{position} \, t=5 - \text{position} \, t=0
\]

Area under graph

\[
= \frac{1}{2} \cdot 1 \cdot 10 + 1 \cdot 10 + 1 \cdot 10 + \frac{1}{2} \cdot 1 \cdot 20 \\
+ 1 \cdot 30 + \frac{1}{2} \cdot 1 \cdot 30 \\
= 5 + 10 + 10 + 10 + 30 + 15 = 80
\]
6. Compute the following

(a) \( \int x \sec^2(x) \, dx \) = \( x \tan(x) - \int \tan^2(x) \, dx \)

Let \( u = x \), \( \frac{du}{dx} = \sec^2(x) \)

\( \frac{du}{dx} = 1 \) \( \Rightarrow \) \( u = \tan(x) \)

\( \Rightarrow \int \tan^2(x) \, dx = x \tan(x) - \int \frac{\sin(x)}{\cos(x)} \, dx \)

\( = x \tan(x) - \int \frac{1}{u} (-du) \)

\( = x \tan(x) + \ln|u| + C \)

\( = x \tan(x) + \ln|\cos(x)| + C \)

(b) \( \int \frac{x^3}{\sqrt{49 - x^2}} \, dx \) = \( \int \frac{17^3 \sin^3 \theta}{7 \cos \theta} \, d\theta \)

Let \( u = \cos \theta \), \( du = -\sin \theta \, d\theta \)

\( \Rightarrow \int \frac{17^3 \sin^3 \theta}{7 \cos \theta} \, d\theta = \int 17 \sin^3 \theta \, d\theta = \int 7 \sin^3 \theta \, d\theta \)

\( = 7^3 \int \sin^3 \theta \, d\theta = 7^3 \int (1 - \cos^2 \theta) \sin \theta \, d\theta \)

\( = -7^3 \int (1 - u^2) \, du \)

\( = -7^3 (u - \frac{u^3}{3}) + C \)

\( = -7^3 \cos \theta + \frac{7^3 \cos^3 \theta}{3} + C \)

\( \Rightarrow \frac{1}{7} \int \frac{17^3}{49 - x^2} \, dx = 7^3 \int \frac{17^3 \sin^3 \theta}{7 \cos \theta} \, d\theta \)

\( = -7^2 \sqrt{49 - x^2} + \frac{1}{3} \sqrt{49 - x^2}^3 + C \)