The following are problems similar to those that may appear on the first mid-term. So doing these problems is a good, but you should also review the homework problems, text, notes, etc.

JUSTIFY ALL ANSWERS. NEATNESS DEFINITELY COUNTS.

1. TRUE or FALSE and GIVE REASON: State if the following is true of false and justify your answer in a sentence (or two at the most).

(a) Suppose you have a sequence $a_1, a_2, \ldots$ that satisfies

$$\lim_{n \to \infty} a_n = 5$$

then

$$\lim_{n \to \infty} a_{2n} = 10.$$ 

(b) Suppose you have a sequence $a_1, a_2, \ldots$ that satisfies

$$\sum_{n=0}^{\infty} a_n$$

converges, then

$$\sum_{n=0}^{\infty} a_{2n}$$

also converges.

2. Compute

$$\lim_{n \to \infty} \frac{n^2 + 5n + 7}{3n^2 - 4n + \ln(n)}$$

3. Determine if the following converge or diverge. If it converges state the value if you can.

(a) 

$$\sum_{n=0}^{\infty} \left(\frac{4}{5}\right)^n$$

(b) 

$$\sum_{n=1}^{\infty} (0.3)^{2n}$$

(c) 

$$\sum_{n=1}^{\infty} \left(\frac{n + 3}{\sqrt{n^6 + 3n + 2}}\right)$$

4. Use the idea behind the integral test to give an estimate on the size of

$$\sum_{n=1}^{100} \frac{n^2}{n^3 + 3}$$

(Be sure to include a figure which helps to explain what you are doing AND be neat!)

5. What is the radius of convergence of
(a) \[ \sum_{n=0}^{\infty} 3nx^n \]

(b) \[ \sum_{n=0}^{\infty} \frac{(x-2)^n}{3n!} \]

6. (a) What is the 5th degree Taylor polynomial for \( \sin(x) \) centered at 0?
   
   (b) What is the maximum error between the 5th degree Taylor polynomial for \( \sin(x) \) centered at 0 and \( \sin(x) \) on the interval \(-2 < x < 2\)?

   (c) What is the maximum error between the 5th degree Taylor polynomial for \( \sin(x) \) centered at 0 and \( \sin(x) \) on the interval \(-1 < x < 1\)?

7. What is the Taylor polynomial of 3rd degree of \( \ln(x) \) centered at \( x = 2 \).

8. The gravitational acceleration on a body near the earth is given by
   
   \[ A(x) = -\frac{GM}{x^2} \]
   
   where \( G \) is the gravitational constant, \( M \) is the mass of the earth and \( x \) is the distance to the center of the earth. Let \( r_0 \) be the radius of the earth.

   (a) Give the Taylor series expansion of degree 3 for \( A(x) \) centered at \( x = r_0 \). (So your answer will have constants \( G \), \( M \) and \( r_0 \)).

   (b) How close is the 2nd degree Taylor polynomial for \( A(x) \) centered at \( r_0 \) to the value of \( A(x) \) on the interval \( r_0 \leq x \leq 2r_0 \)?