3) Solve \((1+x^2)u_y+uy=0\).

We can write the equation as

\[
(1+x^2)u_y+uy=0
\]

or

\[
\nabla u \cdot (1+x^2)=0
\]

That is, at \((x,y)\), the directional derivative

down the direction \((1+x^2)\) is 0.

This defines a vector field on \(\mathbb{R}^2\), let

\[
F(y) = (1+x^2), \quad \rightarrow \quad \nabla u = (1+x^2) - y,
\]

and \(u\) must be constant along curves that are tangent to this direction field, i.e.,

\[
\frac{dx}{dt} = 1+x^2, \quad \frac{dy}{dt} = -1
\]

To write these solution curves on graphs, if \(y\) as a function of \(x\) we need the slope of

solution curves at each point i.e. \(\frac{dy}{dx}\) = change in \(y\) / change in \(x\) = \(\frac{dy}{dt}/\frac{dx}{dt}\).