7) Solve \( a u_x + b u_y + cu = 0. \)

What to do? -- Must try something --

Rewrite

\[ a u_x + b u_y = -c u, \]

or

\[ \nabla U \cdot (\mathbf{g}) = -c u \text{ i.e. in direction } (\mathbf{g}), \]

which looks like exponential decay... if the exponential decay were just in some simple direction --

So let's do a change of coordinates

Let

\[ \tilde{x} = x^*(x,y) = x + B y, \]

\[ \tilde{y} = y^*(x,y) = y + B y. \]

and

\[ \tilde{u}(\tilde{x}, \tilde{y}) = \tilde{u}(x^*(x,y), y^*(x,y)) = u(x,y). \]

Then

\[ u_x = \tilde{u}_x \frac{2B}{x} + u_y \frac{2B}{y} = \tilde{u}_\tilde{x} + \tilde{u}_\tilde{y}, \]

\[ u_y = \tilde{u}_x \frac{2B}{y} + u_y \frac{2B}{x} = \tilde{u}_\tilde{x} \tilde{y} + \tilde{u}_\tilde{y}. \]

If \( a u_x + b u_y + cu = 0 \) then

\[ a(\tilde{u}_\tilde{x} + \tilde{u}_\tilde{y}) + b(\tilde{u}_\tilde{x} + \tilde{u}_\tilde{y}) + cu = 0 \]

\[ (2a + b) \tilde{u}_\tilde{x} + (2b + 2b) \tilde{u}_\tilde{y} + c \tilde{u} = 0 \]