2) $L[u] = \sqrt{1+u^2} \cos y u_x + u u_{xy} - \arctan(y) u.

This is linear

$2[u_{xy}] = \sqrt{1+u^2} \cos y (u w)_x + (u w) u_{xy} - \arctan(y) (u w)_x$

$= \sqrt{1+u^2} \cos y u_x + \sqrt{1+u^2} \cos y u_y$

$+ u x u_y + u y u_x - \arctan(y) u_x + \arctan(y) u_y$

$= 2[u] + 2[w]$

and $2[k w] = \sqrt{1+k^2} \cos y (k u)_x + (k u) u_{xy} - \arctan(y) (k u)_x$

$= k \sqrt{1+k^2} \cos y u_x + k u u_{xy} - \arctan(y) u_x$

$= k 2[u]$. 

3) a) This is 2nd order, linear, non-homogeneous (the $+i$)

b) This is 2nd order, linear, homogeneous

c) This is 3rd order, non-linear (the $w w$ term), homogeneous

d) This is 2nd order, linear, non-homogeneous (the $+i^2$)

e) This is 2nd order, linear, homogeneous.

f) This is 1st order, non-linear ($\sqrt{1+u^2}$), homogeneous

g) This is 1st order, linear, homogeneous

h) This is 4th order, non-linear ($\sqrt{1+u^2}$ term), homogeneous.