7) To check if \(1+x, 1-x\) and \(1+x+x^2\), we ask if we can find constants \(a, b, c\), not all zero such that

\[a(1+x) + b(1-x) + c(1+x+x^2) = 0\] (for all \(x\))

i.e.

\[ax + b - bx + c + cx + cx^2 = 0\]

\[(a + b + c) + (a - b + c)x + c(x^2) = 0\]

Well, to get \(0\) for all \(x\) we need \(c = 0\).

So we consider \((a + b) + (a - b)x = 0\).

If \(x = 0\) then \(a + b = 0\)

If \(x = 1\) then \(a + b + (a - b) = 0\)

So \(2a = 0\) so \(a = 0\)

Since \(a + b = 0\) \(b = 0\)

So the only choice of \(a, b, c\) that gives \(0\) for all \(x\) is \(a = b = c = 0\). So they are independent functions.