Verify \( u_n(x,y) = \sin(my) \sinh(nx) \)

in a solution of \( u_{xx} + u_{yy} = 0 \) for all \( a > 0 \).

Recall \( \sinh(uy) = \frac{e^{uy} - e^{-uy}}{2} \).

More important \( \frac{d}{dx} \sinh(x) = \cosh(x) \)
and \( \frac{d}{dx} \cosh(x) = \sinh(x) \).

So \( \frac{d^2}{dy^2} \sinh(uy) = u^2 \sinh(uy) \).

So \( \frac{d^2}{dx^2} \sinh(uy) = -u^2 \sinh(uy) \sinh(uy) \).

\( \frac{d^2}{dx^2} u_n = u^2 \sinh(uy) \sinh(uy) \)
and \( u_{xx} + u_{yy} = 0 \).

12.1 Solve \( 2u_x + 3u_y = 0 \) with \( u(x,0) = \sin x \).

We seek solutions in the form \( u(t,x) = f(at + bx) \).

Plugging in, we have

\[
2u_t + 3u_x = 2a f'(at + bx) + 3b f'(at + bx)
= (2a + 3b) f'(at + bx)
\]

So we need only choose \( a \) and \( b \) so that

\[
2a + 3b = 0 \quad \text{or} \quad a = -3, b = 2.
\]

In, the characteristics are \(-3t + 2x = \text{constant}\).

\[
t = \frac{3}{2} t + x = \text{constant}
\]

\[
t = -3t = -2x + k \quad \text{or} \quad t = \frac{3}{2}x + k
\]

Any constant.