The Curse of Calculators Or Arithmetic Review

It was clear from the first quiz that calculators have infiltrated pre-college education to a damaging extent. The days when you could earn a living doing arithmetic are long over. However, there is a certain amount of arithmetic that you must be able to do “by hand” or “in your head”.

Reviewing all of your arithmetic is impossible in one document, but you should have the following at your fingertips.

**Estimating Order of Magnitude:** Large errors are generally worse than small errors. The difference between a river being 2 feet or 20 feet over flood stage is very significant, but the difference between 20 and 22 feet over flood stage is not.

The order of magnitude of a number is basically the power of 10 that best approximates it. To say you have computed to the “correct order of magnitude” says you have the decimal point in the right place. If the answer to a problem is 5 and you say it is 0.5 then you are off by an order of magnitude. If the answer is 1003 and you say it is 996 then you have the correct order of magnitude.

There are many ways of estimating order of magnitude—I think it is not a good idea to try to always use the same template. It is much better to be able to think your way through the calculation or order of magnitude and then get quick by practice.

For example, for the fractions

\[
\frac{0.5}{0.2} \text{ and } \frac{5.5}{0.2}
\]

we should go back to basics and say, “how many 0.2’s are needed to make 0.5 or 5.5”. We know

\[
0.5 \approx 0.2 + 0.2
\]

so 0.5/0.2 is approximately 2 or 3. On the other hand we know 1 = 0.2 · 5, so 5.5/0.2 must be 5 times 5 or around 25 to 30.

Similarly

\[
\frac{1.8}{0.5}
\]

is a bit more than 3 because three halves is 1.5 (= 0.5 · 3), just less than 1.8. On the other hand

\[
\frac{1.8}{5}
\]

must be must smaller than 1. It is close to 0.4.

More complicated fractions require a bit more work. For example,

\[
\frac{2847}{17}
\]
can be written
\[
\frac{2847}{17} = \frac{2.847 \cdot 10^3}{1.7 \cdot 10}
\]
we can cancel the 10 in the denominator \(10^3/10 = 10^2\) so
\[
\frac{2847}{17} = \frac{2.847}{17} \cdot 10^2,
\]
so it is close to 150 or 200. Of course, doing this in your head you should just say 2847/17 is like 2000/10 or about 200 or 2847/17 is about 3000/20 or about 150.

You should be able to do this calculation very quickly—that is trade accuracy for speed. You should estimate the order of magnitude for every problem, (even those you do on calculator) and then check your final answer against your estimate. If they don’t agree, stop and see where you made your error.