1. Prove that if \( G \) is a simple group such that \( 61 \leq |G| \leq 70 \), then \( G \) is a cyclic group.  
[Feel free to use the Sylow theorems.]

2. Let \( A \) and \( B \) be normal subgroups of a group \( G \) such that \( A \cap B = \{e\} \). Prove that \( ab = ba \) for all \( a \in A \) and \( b \in B \).  
[Hint: Prove that \( aba^{-1}b^{-1} \in A \cap B \).]

3. Let \( G \) be a group of size 15. Let \( a \) be an element of order 3 and \( b \) be an element of order 5. Set \( A = \langle a \rangle \) and \( B = \langle b \rangle \).  
(a) Prove that \( AB = G \).  
(b) Prove that both \( A \) and \( B \) are normal subgroups.  
[Hint: Use the Sylow theorems.]

4. Let \( G \) be a group of size \( pq \) with \( p \) and \( q \) distinct primes with \( p < q \) and \( q \not\equiv 1 \pmod{p} \). Prove that \( G \) is cyclic.  
[Hint: Mimic the arguments of the last question.]

5. Let \( G \) have size \( 12 = 2^2 \cdot 3 \).  
(a) If \( n_p \) equals the number of \( p \)-Sylow subgroups in \( G \), prove that \( n_2 = 1 \) or \( 3 \) and \( n_3 = 1 \) or \( 4 \).  
(b) Let \( A \) be a 2-Sylow subgroup and let \( B \) be a 3-Sylow subgroup. Prove that either \( A \) or \( B \) is normal.  
[Hint: Otherwise \( n_2 = 3 \) and \( n_3 = 4 \). Show that \( G \) isn’t big enough to have so many \( p \)-Sylows.]

6. (Bonus) Determine all groups of size 12 up to isomorphism.