If 90% of the ideas you generate aren’t absolutely worthless, then you’re not generating enough ideas.
—Michael Artin

Modern Algebra – Math 541 – Fall 2009 – R. Pollack
HW #3

1. Gallian Chapter 2: 3,5

2. Compute $2^{2047423023} \pmod{11}$.

3. Find the multiplicative inverse of 7 in $\mathbb{Z}_{11}$. Find the multiplicative inverse of 7 in $\mathbb{Z}_{101}$.

4. For integers $a$ and $b$, we say $a|b$ (read as $a$ divides $b$) if there exists an integer $k$ such that $ak = b$.
   For each of the following statements either prove them or give a counter-example.
   (a) For all $a$ in $\mathbb{Z}$, we have $a|a$.
   (b) For all $a, b, c$ in $\mathbb{Z}$, if $a|b$ and $b|c$, then $a|c$.
   (c) For all $a, b$ in $\mathbb{Z}$, if $a|b$, then $b|a$.
   (d) For all $a, b, c$ in $\mathbb{Z}$, if $a|b$ and $a|c$, then $a|b + c$.
   (e) For all $a, b, c$ in $\mathbb{Z}$, if $a|b$ and $c|b$, then $a + c|b$.
   (f) For all $a, b$ in $\mathbb{Z}$, if $a|b$, then $a|bc$.
   (g) For all $a, b$ in $\mathbb{Z}$, if $a|bc$, then $a|b$ and $a|c$.

5. Find all $m$ such that $U(m)$ has size 6. How do the multiplication tables of the groups you found compare? Do the same but now find $U(m)$ of size 7.

6. Find all distinct multiplication tables for groups of size 5. (Here “distinct” means, as in class, up to relabeling of the elements.)

7. (Challenge question) Do the same but for groups of size 6!