Complete each of the following exercises.

1. Let $X$ be a set and equip it with the cofinite topology. Prove that $X$ is compact.

2. Let $X$ be a discrete topology space. Find a necessary and sufficient condition on $X$ so that $X$ is compact.

3. Determine whether each of the following subsets of $\mathbb{R}$ are compact under the right-ray topology. Justify your answer:
   (a) $[0, 1]$
   (b) $(0, 1]$
   (c) $[0, 1)$

4. Let $X$ be a topological space with the following property: if $\{F_\alpha\}$ is a collection of closed subsets such any finite intersection of the $F_\alpha$ is non-empty, then $\cap_\alpha F_\alpha$ is non-empty. Prove that $X$ is compact.
   (Hint: take complements and think about open covers.)

5. Let $K$ be a compact subset of $\mathbb{R}^n$ (under the standard topology). Prove that $K$ is closed.
   (Hint: If $K$ is not closed, there is some limit point $x$ of $K$ not in $K$. Show that $\{B_{1/m}(x)^c\}_{m\geq 1}$ is an open cover of $K$ with no finite subcover.)

6. Let $K$ be a compact subset of $\mathbb{R}^n$ (under the standard topology). Prove that $K$ is bounded.
   (Hint: If $K$ were unbounded, then $B_N(0)$ would not contain $K$ for any $N \geq 0$.)

7. Let $X$ be a $T_2$ topological space and let $K_1$ and $K_2$ be compact subsets. Prove that $K_1 \cap K_2$ is compact.
   Does this remain true for infinite intersections?

Additional questions:

1. Does question 7 still hold if we no longer assume that $X$ is $T_2$?

2. Let $K_1$ and $K_2$ be compact subsets of a $T_2$ topology space $X$. Find disjoint open sets $U$ and $V$ such that $K_1 \subseteq U$ and $K_2 \subseteq V$.
   (Hint: use the Theorem proven in class that a point and a compact set can be separated by open sets.)