MA294, Problem Set #3  
Fall 2006

P1: Exercises 20.5, #4

P2. Prove that if \( H \) is a proper subgroup of a finite group \( G \), then \( H \) cannot be isomorphic to \( G \).

P3. Show that the function \( f : \mathbb{Z} \to 2\mathbb{Z} \) given by \( f(n) = 2n \) is an isomorphism. (Thus a proper subgroup of an infinite group may be isomorphic to the whole group.)

P4. Show that the function \( \exp : (\mathbb{R}, +) \to (\mathbb{R}^+, \cdot) \) given by \( \exp(x) = e^x \) is an isomorphism.

P5. Exercises 20.7, #3

P6. Find the center of \( D_7 \), the group of symmetries of the regular 7-gon.

P7. Find the center of \( GL(2, \mathbb{R}) \), the set of \( 2 \times 2 \) invertible matrices with real number entries.