Statistics Seminar Series

Connect-The-Dots: How many random points can a regular curve pass through?

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Friday, February 13, 2004, 4:00-5:00pm
Mathematics and Computer Science (MCS) Building, Room 149
111 Cummingston Street, Boston
Tea and Cookies at 3:30pm in MCS 153

Abstract: Suppose n points are scattered uniformly at random in the unit square. Question: How many of these points can possibly lie on some curve of length bounded by L? Answer, proved here: order the square root of n.

We consider a general class of such questions; in each case, we are given a class \( G \) of curves in the square, and we ask: in a cloud of \( n \) uniform random points, how many can lie on some curve \( g \) in \( G \)? Classes of interest include (in addition to the rectifiable curves mentioned above): Lipschitz curves, increasing curves, twice-differentiable curves, smooth curves with \( m \)-bounded derivatives. In each case we get order-of-magnitude estimates; for example, there are twice-differentiable curves containing as many as order \( n^2 \) of the third uniform random points, but not essentially more than this.

We also consider generalizations to higher dimensions and to hypersurfaces of various co-dimensions. Thus, twice-differentiable \( k \)-dimensional hypersurfaces in dimension \( d \) may contain as many as \( n \) to the power \( k/(2d-k) \) uniform random points. We also consider other notions of 'passing through' such as passing through given space/direction pairs. Thus, twice-differentiable curves in 2D may pass through at most order \( n \) to the fourth uniform random points.

We give both concrete approaches to our results, based on geometric multiscale analysis, and abstract approaches, based on epsilon-Entropy.

Stylized applications in image processing and perceptual psychophysics are described.

For directions and maps, please see http://math.bu.edu/research/statistics/statseminar.html. For other information, please contact Eric Kolaczyk (kolaczyk@math.bu.edu) or the main department office at (617)353-2560.)