Exponential Dichotomies in Spatial Evolutionary Systems for Elliptic PDEs

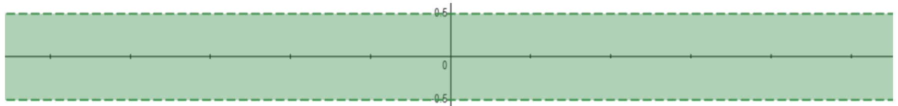
Consider an evolutionary equation:

$$u_x = A(x)u, x \in \mathbb{R}, u \in X$$

Exponential dichotomies **describe the behavior of exponentially decaying solutions**, even when solutions generally don't exist.

With them, we can **construct bounded solutions**, for example, solutions that represent interesting patterns that arise in nature.

Spatial dynamics describes a perspective of studying PDEs which **views spatial variables as evolutionary variables**.



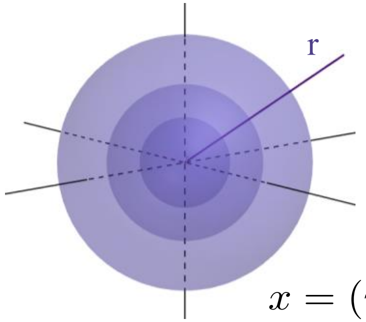
$$x = (x_1, x_2) \in \Omega = \mathbb{R} \times (-\tfrac{1}{2}, \tfrac{1}{2})$$

Elliptic PDE in channel:

$$u_{x_1 x_1} + u_{x_2, x_2} + a(x)u = 0$$

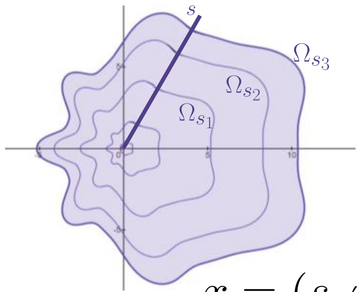
↓

$$\mathbf{u}_{x_1} = \mathcal{A}(\partial_{x_2}, x_1, x_2)\mathbf{u}$$



$x = (r, \varphi)$

$\mathbb{R}^n = [0, \infty) \times S^{n-1}$



$x = (s, \omega)$

$D = [0, \infty) \times \Omega_s$

Elliptic PDE in $\mathbb{R}^n$:

$$\Delta u = V(x)u$$

↓

$$\mathbf{u}_r = \mathcal{A}(\partial_\varphi, r, \varphi)\mathbf{u}$$

Theorem: If *V(x)* decays to a constant like 1/x and is Hölder continuous for *r<1*, then **the above spatial dynamical system for the elliptic PDE has an exponential dichotomy** for all *r*.