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Today's topics

1 Course introduction

- Please refer to the syllabus for all course details.
- For everyone's health and safety, please **do not** come to class if you come down with flu-like or cold-like symptoms! Preventing the spread of COVID-19 depends on us all doing our part. If you miss class due to illness, let me and the TF for your discussion section know and we will do our best to help you catch up with anything you missed.
- Please make sure that your facemask is well-fitted and is covering your nose and mouth at all times while in the classroom.
- We will use MyLab Math for homework. Create your MyLab Math account using your BU email address and BU ID number (in the format `U12345678`). Our course code is `hast69411`. You can register for 14 days without having to pay; if you do that, make sure you upgrade your original account rather than starting from scratch.
- How lectures will work:
  - Notes will be available online (https://math.bu.edu/people/drhast/MA123-f21/MA123-f21.html) for use in each class.
  - We will sometimes use Learning Catalytics (through MyLab Math).
  - Examples will usually be even-numbered exercises from the textbook.
- Chapter 1 of the textbook gives a review of functions (definitions of various functions: polynomials, rational, exponential, logarithmic, trigonometric; transformations of functions, inverse functions, etc.), all of which will play a crucial role throughout our semester.
  - It is very important to be familiar with the content of this foundational first chapter. Please take a look and do some practice problems this week if any of the concepts are unfamiliar or a bit hazy after the summer.
  - You will have a review worksheet covering some of the topics in Chapter 1 during the first discussion session.

  - Let your TF know if you have not covered a specific topic.
  - We will also review some of the more challenging topics (e.g., inverse trig functions) in lecture when they show up.

- Calculus I covers limits, differentiation, and integration (chapters 2–5 of the textbook).

2 Big-picture overview of calculus

- Two key operations: **differentiation** (derivatives) and **integration** (integrals).
- **Differentiation:** Given a function, compute the *rate of change* at each point. For example:
  - If we know distance traveled over time, find the velocity.
  - Given velocity, find the acceleration.
  - Given amount of water in a container at each point in time, compute rate of inflow or outflow.
- **Integration:** Given a function, compute the *area under the curve.* For example:
  - Given velocity, compute distance traveled.
  - If we know acceleration, use it to find velocity.
  - Given the rate of flow of water into a container, find the total amount of water added to the container.
- These processes *reverse each other*! We will come back to this later in the course.
- To reason about differentiation and integration, we need the fundamental concept of **limits**.

3 The idea of limits: average/instantaneous velocity

Briggs–Cochran–Gillett §2.1 pp. 54–60

3.1 Motivating example: average velocity

2016 Summer Olympics 100m men's race: final results

	Distance	Time
Bolt	100m	9.81s
Gatlin	100m	9.89s

How fast did they go?

Recall: If $s(t)$ is the position at time t, the *average velocity* between t_0 and t_1 is

$$\boxed{v_{av} = \frac{s(t_1) - s(t_0)}{t_1 - t_0}}.$$

Bolt's average velocity from start to finish:

$$v_{av} = \frac{s(t_{\text{end}}) - s(t_{\text{start}})}{t_{\text{end}} - t_{\text{start}}} = \frac{100 \text{ m}}{9.81 \text{ s}} \approx 10.19 \text{ m/s}$$

The race: https://www.youtube.com/watch?v=4gUW1JikaxQ&t=190

Graphs of position at various times:

Gatlin clearly started faster (covered more distance in the same amount of time), but then Bolt caught up. Their velocities changed during the race.

3.2 From average to instantaneous velocity, geometric idea

Example 1 (§2.1 **Ex. 10**) *The position of an object moving vertically along a line is given by the function* $s(t) = -4.9t^2 + 30t + 20$. *Find the average velocity of the object in the intervals* $[0, 3]$, $[0, 2]$, $[0, 1]$ *and* $[0, h]$ *where* $h > 0$ *is a real number.*

Solution: Let $h > 0$ be a real number. Then the average velocity in the interval $[0, h]$ is

$$\begin{aligned} v_{av} &= \frac{s(h) - s(0)}{h - 0} \\ &= \frac{(-4.9h^2 + 30h + 20) - 20}{h - 0} \\ &= \frac{-4.9h^2 + 30h}{h} \\ &= -4.9h + 30. \end{aligned}$$

Example 2 (§2.1 **Ex. 28**) *Let $f(x) = x^3 - x$. Make a table of slopes of secant lines and make a conjecture about the slope of the tangent line at $x = 1$. (Recall that a* ***secant line*** *is a straight line joining two points on a curve.)*

For the interval $[1, t]$, the slope of the secant line is

$$\frac{(t^3 - t) - (1^3 - 1)}{t - 1} = \frac{t^3 - t}{t - 1}.$$

Here's one possible table (you could choose other endpoints):

Interval	Slope of secant line
$[1, 2]$	6
$[1, 1.5]$	3.75
$[1, 1.1]$	2.31
$[1, 1.01]$	2.0301
$[1, 1.001]$	2.003001

This strongly suggests that the slope of the tangent line at $x = 1$ is 2.

To summarize:

The instantaneous velocity at $t = t_0$	=	"Limit as $t \to t_0$ of the average velocities in the intervals $[t_0, t]$"

This geometrically corresponds to the following:

The slope of the tangent line to $s(t)$ at $(t_0, s(t_0))$	=	"Limit as $t \to t_0$ of the slopes of the secant lines between $(t_0, s(t_0))$ and $(t, s(t))$"