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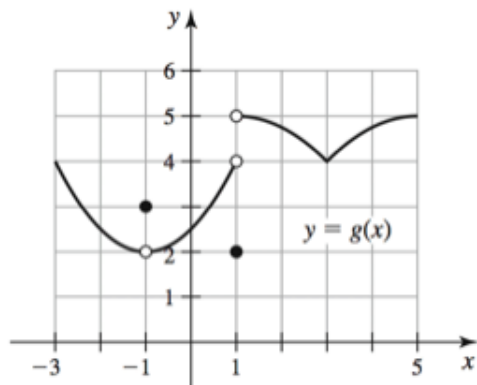
Today's topics

1 One-sided limits: example	1
2 Techniques for computing limits	2
2.1 Limit laws	2

Note: the first homework assignment is up on MyLab Math. If you haven't already done so, please sign up for MyLab Math (with your BU email, BU ID, and the course code hast69411).

1 One-sided limits: example

Example 1 Use the graph of g in the figure to find the following values or state that they do not exist. If a limit does not exist, explain why.



- | | | |
|--------------------------------------|------------------------------------|-------------------------------------|
| (a) $g(-1)$ | (d) $\lim_{x \rightarrow -1} g(x)$ | (g) $\lim_{x \rightarrow 3} g(x)$ |
| (b) $\lim_{x \rightarrow -1^-} g(x)$ | (e) $g(1)$ | (h) $g(5)$ |
| (c) $\lim_{x \rightarrow -1^+} g(x)$ | (f) $\lim_{x \rightarrow 1} g(x)$ | (i) $\lim_{x \rightarrow 5^-} g(x)$ |

2 Techniques for computing limits

Briggs-Cochran-Gillett §§2.3 – 2.4 pp. 69–88

2.1 Limit laws

Here are some very helpful limit laws which let us simplify the evaluation of many limits:

Theorem 2 (Limit laws) Assume $\lim_{x \rightarrow a} f(x)$ and $\lim_{x \rightarrow a} g(x)$ exist. The following properties hold, where c is a real number and n is a positive integer:

1. **Sum:** $\lim_{x \rightarrow a} (f(x) + g(x)) = \lim_{x \rightarrow a} f(x) + \lim_{x \rightarrow a} g(x)$
2. **Difference:** $\lim_{x \rightarrow a} (f(x) - g(x)) = \lim_{x \rightarrow a} f(x) - \lim_{x \rightarrow a} g(x)$
3. **Constant multiple:** $\lim_{x \rightarrow a} (cf(x)) = c \lim_{x \rightarrow a} f(x)$
4. **Product:** $\lim_{x \rightarrow a} (f(x)g(x)) = \left(\lim_{x \rightarrow a} f(x) \right) \left(\lim_{x \rightarrow a} g(x) \right)$
5. **Quotient:** $\lim_{x \rightarrow a} \left(\frac{f(x)}{g(x)} \right) = \frac{\lim_{x \rightarrow a} f(x)}{\lim_{x \rightarrow a} g(x)}$, provided $\lim_{x \rightarrow a} g(x) \neq 0$
6. **Power:** $\lim_{x \rightarrow a} (f(x))^n = \left(\lim_{x \rightarrow a} f(x) \right)^n$
7. **Root:** $\lim_{x \rightarrow a} (f(x))^{1/n} = \left(\lim_{x \rightarrow a} f(x) \right)^{1/n}$, provided that $f(x) \geq 0$ for all x near a if n is even.

Example 3 Assume $\lim_{x \rightarrow 1} f(x) = 8$, $\lim_{x \rightarrow 1} g(x) = 3$, and $\lim_{x \rightarrow 1} h(x) = 2$. Compute the following limits and state the limit laws used to justify your computations.

(a) $\lim_{x \rightarrow 1} \frac{f(x)}{h(x)}$

(b) $\lim_{x \rightarrow 1} (f(x)h(x))$

(c) $\lim_{x \rightarrow 1} \sqrt[3]{f(x)g(x) + 3}$

We record a few special cases (really, just applications of the limit laws above), limits of polynomial functions and rational functions:

Theorem 4 *Assume p and q are polynomials and a is constant.*

1. **Polynomial functions:** $\lim_{x \rightarrow a} p(x) = p(a)$

2. **Rational functions:** $\lim_{x \rightarrow a} \frac{p(x)}{q(x)} = \frac{p(a)}{q(a)}$, provided $q(a) \neq 0$.

Example 5 *Evaluate the following limits:*

(a) $\lim_{t \rightarrow -2} (t^2 + 5t + 7)$

(b) $\lim_{t \rightarrow 3} \sqrt[3]{t^2 - 10}$