Dr. Daniel Hast, *drhast@bu.edu*

# Today's topics



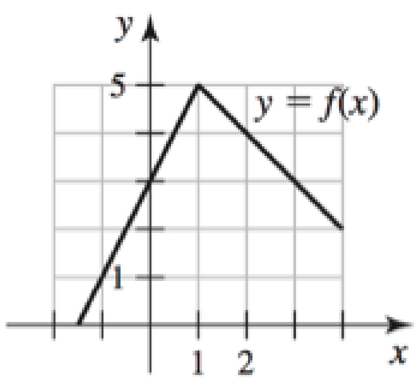
# 1 Derivatives

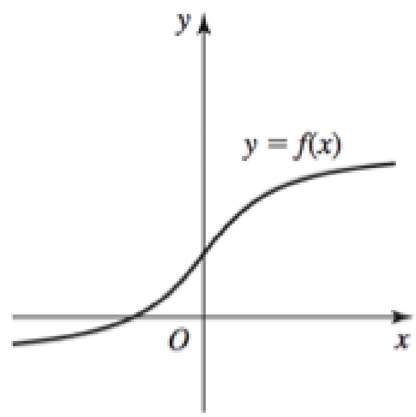
Briggs–Cochran–Gillett §3.2, pp. 140–152

## 1.1 Graphs of functions and their derivatives

Having defined the derivative, we now explore how the graphs of a function and its derivative are related. (You will be expected to recognize the derivative of a function given the graph of the function.)

**Example 1.** For each of the following functions $f$, use the graph of $f$ to sketch a graph of $f'$:



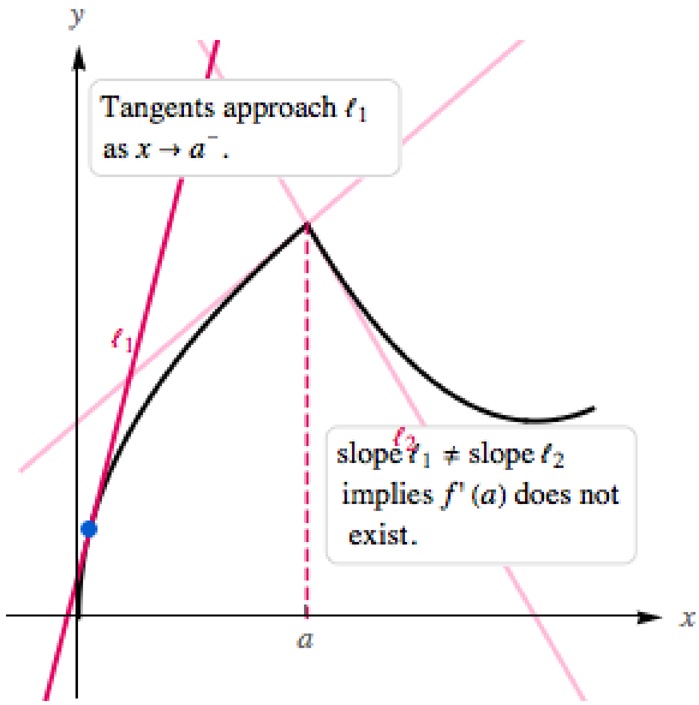

## 1.2 Differentiability and continuity

**Theorem 2** (Differentiable implies continuous)**.** *If $f$ is differentiable at $a$, then $f$ is continuous at $a$.*

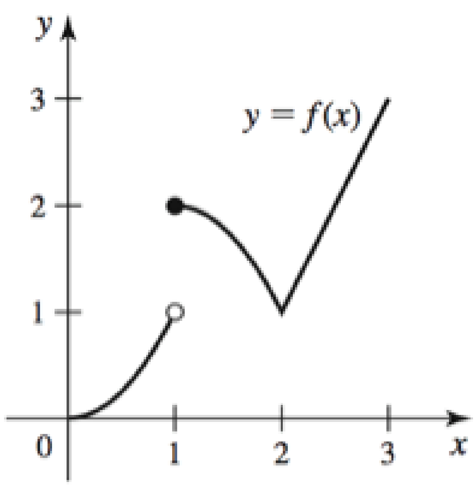
This can be stated in another way:

**Theorem 3** (Not continuous implies not differentiable)**.** *If $f$ is not continuous at $a$, then $f$ is not differentiable at $a$.*

Be careful: it might be tempting to read more into Theorem 2 than what it actually states. Note that if $f$ is continuous at a point, $f$ is *not necessarily* differentiable at that point. Here is one such example:




**Example 4.** Use the graph of $f$ in the figure to do the following.




1. Find the values of $x$ in (0,3) at which $f$ is not continuous
2. Find the values of $x$ in (0,3) at which $f$ is not differentiable.
3. Sketch a graph of $f'$.

# 2 Rules of differentiation

Briggs–Cochran–Gillett §3.3, pp. 152–163

## 2.1 The constant, power, constant multiple, and sum rules

**Theorem 5** (First Differentiation Rules). *Let $c$ be a constant, $n$ a positive integer and $f$ and $g$ differentiable functions.*

- *Constant rule:* $\dfrac{d}{dx}(c) = 0$.
- *Power rule:* $\dfrac{d}{dx}(x^n) = nx^{n-1}$.
- *Constant multiple rule:* $\dfrac{d}{dx}(cf(x)) = cf'(x)$.
- *Sum rule:* $\dfrac{d}{dx}(f(x) + g(x)) = f'(x) + g'(x)$.

**Example 6.** Find the derivative of $g(x) = 6x^5 - x$.

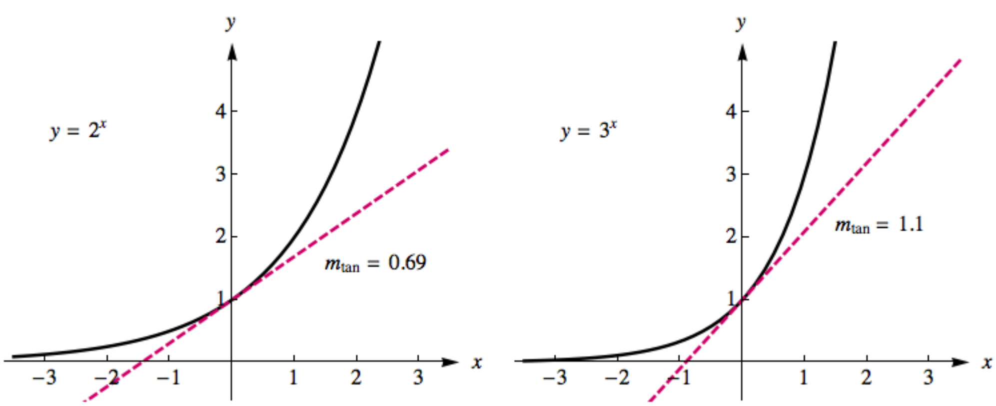
**Example 7.** Find the derivative of $f(t) = 6\sqrt{t} - 4t^3 + 9$.

**Example 8.** Find the derivative of $g(r) = (5r^3 + 3r + 1)(r^2 + 3)$ by first expanding the expression. Simplify your answer.

## 2.2 Derivative of $e^x$

**Definition of $e$**

Exponential functions $b^x$ look like



The number $e$ can be defined as the base needed in the exponential function to get the slope of the tangent to the graph at $x = 0$ equal to 1. We have $2.7182 < e < 2.7183$.

**Definition 9.** *$e^x$ is the exponential function such that the slope of the tangent to the graph at $x = 0$ is 1, i.e.,*

$$\lim_{h \to 0} \frac{e^h - 1}{h} = 1.$$

**Derivative of $e^x$**

**Theorem 10.** *The function $f(x) = e^x$ is differentiable for all real numbers $x$, and*

$$\frac{d}{dx} e^x = e^x.$$

**Example 11.** Find an equation of the tangent line to $y = \dfrac{e^x}{4} - x$ at $a = 0$. Then use a graphing utility to graph the curve and the tangent line on the same set of axes.