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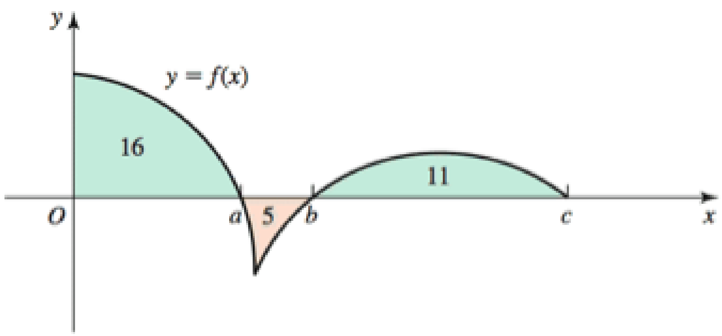
Today's topics

1 Definite integrals

Briggs–Cochran–Gillett §5.2 pp. 353–367

1.1 Evaluating definite integrals

Example 1 (§5.2 Ex. 59, 60, 61, 62)**.** The figure shows the areas of regions bounded by the graph of f and the x-axis. Evaluate the following integrals.



1. $\int_0^a f(x)dx$
2. $\int_0^b f(x)dx$
3. $\int_a^c f(x)dx$
4. $\int_0^c f(x)dx$

We can use familiar area formulas from geometry to evaluate certain definite integrals.

Example 2. Use geometry (not Riemann sums) to evaluate the definite integral

$$\int_{-1}^{3} \sqrt{4-(x-1)^2}dx.$$

Sketch a graph of the integrand, show the region in question, and interpret your result.

(Hint: Observe that the graph is the upper half of a circle of radius 2 centered on $(1,0)$.)

We can also write down Riemann sums, take the limit as $n \to \infty$, and use the formulas for sums of powers of integers to compute certain definite integrals.

Example 3 (§5.2 Ex. 80, 82)**.** Use the definition of the definite integral to evaluate the following definite integrals. Use right Riemann sums and results on sums of powers of integers.

1. $\int_{1}^{5}(1-x)dx$

2. $\int_{0}^{2}(x^2-1)dx$

1.2 Properties of definite integrals

We first establish some criteria for a function to be integrable:

Theorem 4 (Integrable functions)**.** *If f is continuous on $[a, b]$ or bounded on $[a, b]$ with a finite number of discontinuities, then f is integrable on $[a, b]$.*

Here are some very important properties of definite integrals:

Let f and g be integrable functions on an interval that contains a, b, and p.

1. $\int_a^a f(x)\,dx = 0$ Definition

2. $\int_b^a f(x)\,dx = -\int_a^b f(x)\,dx$ Definition

3. $\int_a^b (f(x) + g(x))\,dx = \int_a^b f(x)\,dx + \int_a^b g(x)\,dx$

4. $\int_a^b c\,f(x)\,dx = c\int_a^b f(x)\,dx$ For any constant c

5. $\int_a^b f(x)\,dx = \int_a^p f(x)\,dx + \int_p^b f(x)\,dx$

6. The function $|f|$ is integrable on $[a, b]$ and $\int_a^b |f(x)|\,dx$ is the sum of the areas of the regions bounded by the graph of f and the x-axis on $[a, b]$.

Example 5 (§5.2 Ex. 52)**.** Suppose $\int_1^4 f(x)dx = 8$ and $\int_1^6 f(x)dx = 5$. Evaluate the following integrals.

1. $\int_1^4 (-3f(x))dx$
2. $\int_1^4 3f(x)dx$
3. $\int_6^4 12f(x)dx$
4. $\int_4^6 3f(x)dx$

Example 6 (§5.2 Ex. 54)**.** Suppose $f(x) \geq 0$ on $[0, 2]$, $f(x) \leq 0$ on $[2, 5]$, $\int_0^2 f(x)dx = 6$, and $\int_2^5 f(x)dx = -8$. Evaluate the following integrals.

1. $\int_0^5 f(x)dx$
2. $\int_0^5 |f(x)|dx$
3. $\int_2^5 4|f(x)|dx$
4. $\int_0^5 (f(x) + |f(x)|)dx$