# Math 876: PDE Seminar Spring 2023

## Course Info

**Instructor:** Jonathan Jaquette

**Times:** Tue, Thu9:30am 10:45am

**Location:** CGS 111A (871 Commonwealth Avenue, Boston MA)

**Office Hours:** In MCS 273 on Monday & Wednesday (at a time TBD), and by appointment

**Required Textbooks:**

- Boyd, John P. *Chebyshev and Fourier spectral methods*. Courier Corporation, 2001. 2nd Edition
- Trefethen, Lloyd N. *Spectral methods in MATLAB*. Society for industrial and applied mathematics, 2000.

**Course Website:** Apart from the textbook, all course readings, announcements, and supplemental materials will be posted through the course Blackboard page.

## Course Content

What is the "best" way to approximate a function? Taylor series can provide a great local representation but may diverge outside a disappointingly small radius of convergence, as with the witch of Agnesi. In contrast, Fourier series and other spectral methods have been developed to provide a globally convergent approximation, and are often exceptionally efficient at doing so. The goal of this course is to study spectral methods for problems arising in ordinary and partial differential equations, such as solving boundary value problems and finding eigenvalues/eigenfunctions

## Course Objectives

By the end of the course, students will be able to:

- Numerically solve problems in differential equations (eg IVP, BVP, finding eigenvalues/eigenfunctions) using spectral methods in MATLAB.
- Critically read the textbook and journal articles, summarize the major points, and discuss the finer details.
- Prepare and deliver 15-20 minute presentations on PDEs & numerical methods.

## Course Pre-Requisites:

The standard pre-requisite for this class is to have taken (or be currently taking) *MA 776 Partial Differential Equations*. Alternatively, if you have taken some number of the classes below, then you are probably well prepared for MA 876.

MA 711 Real Analysis; MA 713 Functions of a Complex Variable; MA 717 Functional Analysis; MA 771 Introduction to Dynamical Systems; MA 775 Ordinary Differential Equations

If you have any questions or concerns about your preparation for this course, come talk to me about it!

## Seminar Format

This course is a seminar and, unlike a traditional course, almost all of the talking in our meetings is done by the students. The main work of the seminar meeting will be presentations on selected topics, discussion of the readings and presentations, discussion of student generated questions, and presentation and discussion of homework problems. I'd like the seminar atmosphere to be lively but not intimidating. The goal of the group should be to push everyone's understanding. If something isn't clear, we need to stop and clarify.

In this format, the seminar members will be expected to have mastered all the basic background material before seminar, and the seminar meeting will be devoted to reinforcing, extending and enhancing your knowledge of the seminar topics. Students presenting material in any given week must thoroughly prepare those presentations in advance. Students not presenting material should still study all of the material to be presented in advance so that they may ask good questions and participate fully in the discussions. If you are unable to attend class for any reason, please contact me as soon as possible.

Many of us are probably new to the seminar format. Please feel free to make suggestions of things we can do differently that will help you learn better along the way.

### Meeting Organization

Below is an example agenda for one week's seminar:

Tuesday Class:

- Brief summary of the week's reading (5 min)
- Student presentations (40-50 min)
- Discussion of students' prepared questions on the reading (20-30 min)

Thursday Class:

- Plan which problems to discuss (5 min)
- Discussion of homework problems (60 min)
- Closing, and planning for the next week (5-10 min)

### Discussion Questions:

Each week there will be about 25 pages from the book that everyone should read by Tuesday.

At least 1 hour before class on Tuesday, each student must email me a list of **at least 3 questions** they had from the reading. My suggestion would be to compile your list of questions as you are reading the week's material.

In class on Tuesday we will go around the room. Each person will pose one of their questions and the group will discuss. If there is time remaining we will discuss any further questions/topics the group finds to be of interest.

## Presentations

Many seminars will include a short presentation or two prepared by students. I'll ask for volunteers for presentations the week before. Presentations may include discussing a section from our text or another book, readings from journals, examples relating to the week's reading, or numerical demonstrations.

Each presentation should be 15-20 minutes and leave 5-10 minutes for questions. You should not go overtime; I will set a timer. Depending on the size of the class, each student may expect on average to give a presentation every other week.

In weeks that you are giving a presentation, you should talk to me in office hours before hand to discuss your talk outline/ask questions about the material/etc. I will also give you feedback on

## Homework Problems

Each week there will be a handful of homework problems to accompany the reading. You are strongly encouraged to work together on problems. While you will not be turning in a write up for these problems, you are expected to attempt as many problems as you can and come to class prepared to discuss them.

At the beginning of each Thursday seminar we'll decide who will lead the discussion on which homework problems. You'll let me know the problems you feel most comfortable with and I'll respect your preferences. Difficult problems we can all work on together.

## Tentative Schedule of Material

| Week | Tues | Thurs | Topic | Reading |
|---|---|---|---|---|
| 1 | --- | 1/20 | Introduction to Spectral Methods | B1*, T1** |
| 2 | 1/25 | 1/27 | Chebyshev and Fourier Series | B2, T2-3 |
| 3 | 2/1 | 2/3 | Chebyshev and Fourier Series | B2, T4 |
| 4 | 2/8 | 2/10 | Galerkin & Weighted Residual Methods | B3, T12 |
| 5 | 2/15 | 2/17 | Interpolation & Collocation | B4, T5-6 |
| 6 | --- | 2/24 | Cardinal Functions | B5 |
| 7 | 3/1 | 3/3 | Pseudospectral Methods for BVP | B6, T7-8 |
| | | | Spring Break | |
| 8 | 3/15 | 3/17 | Linear Eigenvalue Problems | B7, T9 |
| 9 | 3/22 | 3/24 | Time-Integration Methods | B9, T10 |
| 10 | 3/29 | 3/31 | Partial Summation, the FFT & MMT | B10 |
| 11 | 4/5 | 4/7 | Coordinate Transformations | B16 |
| 12 | 4/12 | 4/14 | Methods for Unbounded Intervals | B17 |
| 13 | 4/19 | 4/21 | Spherical & Cylindrical Geometry | B18, T11 |
| 14 | 4/26 | 4/28 | Validated Numerics & Computer Assisted Proofs | TBD |
| 15 | 5/3 | --- | Open Topic | TBD |

*B1 – Boyd Chapter 1; **T1 -- Trefethen Chapter 1

## Numerics

Programing will be an integral part of the course. We will use MATLAB as a common programming language in this class, which as a BU graduate student you may download for free. If you need any help with programming, please come to office hours for help!

## Open Topics

I have left the topics for the last three weeks open for the class to decide on. For these weeks I propose we move away from the text book and read seminal research papers on dynamics of PDEs. Some possible topics could include: bifurcations, chaos, computer assisted proofs, inertial manifolds, and/or other types of evolutionary equations (eg. delay differential equations, integro-differential equations). I am very open to other suggestions you might have. Around spring break, I will come up with a list of possible papers and we will finalize the topics for the last three weeks.

## Course Grades

The grade will be A for those students who remain involved with the course throughout the semester and contribute to its success.


## Acknowledgements

Thank you to Jennifer Balakrishnan and my professors in the math & physics departments at Swarthmore College for being an inspiration in designing this course.