

APMA 1680: Nonparametric Statistics

Division of Applied Mathematics, Brown University, Spring 2010

Instructor: Konstantinos Spiliopoulos.

Homework 1: Some warm up problems.

Due: Thursday Feb the 11th.

1. Seven boys and 10 girls take an examination and each student has probability of 0.2 of failing the examination.

1. What is the sample space for this experiment?
2. Given that three students failed the examination, what is the probability that all three are boys?
3. If the probability of each failure is 0.8 instead of 0.2, what will your answer be?

2. If $P(X = 0, Y = 0) = 1/8$, $P(X = 0, Y = 1) = 3/8$, $P(X = 1, Y = 0) = 3/8$ and $P(X = 1, Y = 1) = 1/8$ find: $E(X)$, $E(Y)$, EX^2Y , $cov(X, Y)$, $P(X = x)$ for all x . Are X and Y independent?

3. If Y is a binomial random variable with parameters $n = 60, p = 1/2$, estimate the probability that the random variable $\frac{(Y-np)^2}{np(1-p)}$ will exceed 5.

4. Let W be a chi-square random variable with k degrees of freedom. Find the 0.95 quantile of W if $k = 200$.

5. Let X be the number of consumers who visit the Dairy Queen in Plains, Texas, at least one once during October. Assume that there are 2000 people living in Plains and each has probability of 0.25 of visiting the Dairy Queen, independent of one another. Find $P(460 < x < 540)$.

6. A television station asks the question "Should liquor by the drink be allowed in Kansas" and reports 372 phone calls, in which 164 persons said "no" and the remainder said "yes".

1. What was the target population?
2. What was the sampled population?
3. Was the sample a random sample? Explain.
4. Three statistics are indicated in this exercise. What are they and what numerical values did they assume?

7. Ten persons are selected at random from among all persons living in a particular community. The taxable incomes for five of these persons in the previous calendar year were \$8600, \$15,200, \$16,200, \$16,400, \$29,600. There was no income for the other 5 people.

1. Draw the graph of the empirical distribution function.
2. Find the sample mean income.
3. Find the sample median income.
4. Find the sample variance and sample standard deviation.

8. A new teaching method is being tested to see if it is better than the existing one.

1. What are the appropriate H_0 and H_1 ?
2. What does "level of significance" represent in this problem ?
3. What does "power" represent in this problem ?

10. The hypothesis $H_0 : p = 1/2$ is being tested against $H_1 : p = 3/4$ using two tests T_1 and T_2 at the same level of significance. If T_1 uses a sample of size 20, then T_2 requires a sample of size 35 in order for the power of T_2 to equal the power of T_1 .

1. What is the efficiency of T_2 relative to T_1 ?
2. What is the efficiency of T_1 relative to T_2 ?